Problem 1 (Stability Robustness Analysis - Delay Margin)
Consider the LTI plant \( P = P_0 e^{-\delta s} \) where \( P_0 = \frac{1}{s+1} \) and \( \delta > 0 \). Consider the nominal controller \( K = 1 \).
(a) Determine the delay margin of the nominal design.
Now we wish to use the small gain condition to estimate the delay margin. Do so using a (b) multiplicative modeling error characterization, (c) additive modeling error characterization. Comment on the conservatism of the answers obtained in (b) and (c) with respect to the exact answer in (a).

Problem 2 (Stability Robustness Analysis - High Frequency Dynamics)
Consider the LTI plant \( P = P_0 \left[ \frac{100}{s+100} \right] \left[ \frac{\omega_n^2}{s^2 + 2(0.7651)s + \omega_n^2} \right] \) where \( P_0 = \frac{1}{s+1} \) and \( \omega_n > 0 \). Consider the nominal controller \( K = \frac{3(s+\delta)}{s} \cdot \frac{100}{s+100} \). (a) Determine how small \( \omega_n \) can be made before instability sets in.
Now we wish to use the small gain condition to estimate the minimum possible \( \omega_n \). Do so using a (b) multiplicative modeling error characterization, (c) additive modeling error characterization. Comment on the conservatism of the answers obtained in (b) and (c) with respect to the exact answer in (a).
(d) Show and discuss how \( L = PK, S = \frac{1}{1+PK}, T = I - S, KS, \) and \( SP \) change as \( \omega_n \) is reduced. Choose 3 \( \omega_n \) values to generate your Bode plots (magnitude and phase for \( L \), magnitude for others). Note: Your \( S \) plot should have 3 curves on it. Similar for other plots.

Problem 3 (Stability Robustness Analysis - PUMA Robotic Manipulator)
Let \( P = P_0 \left[ \frac{100}{s+100} \right] \left[ \frac{\omega_n^2}{s^2 + 2(0.7651)s + \omega_n^2} \right] \) where \( P_0 \) represents the two-input two-output transfer function matrix for the PUMA robotic manipulator in your text and \( \omega_n > 0 \). Consider the nominal \( H^\infty \) design discussed in class. (a) Determine how small \( \omega_n \) can be made before instability sets in.
Now we wish to use the small gain condition to estimate the minimum possible \( \omega_n \). Do so using a (b) multiplicative modeling error characterization at the plant output, (c) additive modeling error characterization at the plant output. Comment on the conservatism of the answers obtained in (b) and (c) with respect to the exact answer in (a).
(d) Show and discuss how \( L = PK, S = (I + PK)^{-1}, T = I - S, KS, \) and \( SP \) change as \( \omega_n \) is reduced. Provide 3 representative sets of singular value plots to support your discussion.

Problem 4 (LQR Design for Modified F8 Aircraft)
Do LQR design problem for modified F8 in text. Discuss how you might improve the design?

Problem 5 (\( H^\infty \) Design for Modified F8 Aircraft)
Try to obtain a comparable \( H^\infty \) design to the LQR design. Compare your \( H^\infty \) design with the LQR design in terms of command following, disturbance attenuation, and noise attenuation.

Problem 6 (Control System Design for Fishery)
Consider the following simplified Gordon-Schaefer model for a fishery:

\[
\dot{x} = r x \left(1 - \frac{x}{k}\right) - q x u_p, \quad x(0) = x_o
\]

where \( x \) represents quantity of fish, \( r > 0 \) represents a growth parameter, \( k > 0 \) represents carrying capacity or maximum number of fish, \( q \) represents a catchability parameter, \( u_p \in [0,1] \) represents harvesting effort, and \( t \) is time (measured in years).
(a) Equilibria. Determine all system equilibria \((x_e, u_{pe})\).
(b) Linearization. Determine a linear model \( P = [A, B, C, D] \) about \((x_e, u_{pe})\).
Nominal Parameters. Suppose that \( r = q = 0.3, k = 100, x_o = 18.75, x_e = 37.5014 \). Show that \( u_{pe} = 0.6250 \).
Control System Structure. Consider the control system structure \( K = \frac{g(s+\epsilon)}{s+2} \) (PI controller), \( u = Ke, e = Wxd - x, W = \frac{1}{s+2} \) (command pre-filter), \( x_d = x_e \) is the desired fish biomass, and

\[
u_p = sat(u) \begin{cases} 
0 & u < 0 \\
1 & u > 1 \\
u & 0 \leq u \leq 1
\end{cases}
\]

The above nonlinearity is easily implemented within Simulink.
(c) **Design Based On Constrained Linear System.** Apply the above control system structure to the following constrained linear system: \( \delta y = \delta y_p + x_e, \delta y_p = P \delta u_p, \delta u_p = sat(u) - u_e, \delta x(0) = x_o - x_e. \) Here, \( x_e \) can be thought of as a disturbance at the linear plant output while \( -u_e \) can be thought of as a disturbance at the plant input. Justify the use of the above constrained linear model. Determine design parameters \( g \) and \( z \) such that the overshoot in the output for the above feedback loop is about 0.5580\% and the settling time is about 8-9 years. NOTE: Some saturation will take place initially. Plot all relevant frequency responses for your design. Discuss its properties.

(d) **Nonlinear Simulation.** When your linear design is applied directly to the nonlinear system (i.e. \( K = \frac{g(s+z)}{s} \) (PI controller), \( u = Ke, e = W x_d - x, W = \frac{z}{s+z} \) (command pre-filter), \( x_d = x_e \) is the desired fish biomass, \( u_p = sat(u), x(0) = x_o \)), the resulting response should result in at most 3.0440\% overshoot with about 11 years settling time. Plot the resulting \( x, u_p, \) and \( u \) obtained from the nonlinear system together with the corresponding time responses obtained using the constrained linear system.

NOTE:
All work should be documented within a (very neat) comprehensive report.
Ambiguous answers or sloppy work will not be graded.
(A type written report is NOT required NOR suggested!)