Problem 1 (Dynamic Augmentation: State Space Representation)
Consider a dynamical system consisting of several subsystems. The describing equations are as follows:
\[ y_1 = x_{a_1}, \quad \dot{x}_{a_1} = y_{p_1} 
\]
\[ y_2 = y_{p_2} 
\]
\[ y_p = C_p x_p + D_p u_p, \quad \dot{x}_p = A_p x_p + B_p u_p 
\]
\[ u_p = u_1, \quad u_{p_2} = x_{a_2}, \quad \dot{x}_{a_2} = -x_{a_2} + u_2 
\]
Sketch a block diagram and determine a state space representation for the above dynamical system.

Problem 2 (Gaussian Elimination, Fundamental Spaces, Least Squares, Minimum Norm)
Consider the following linear algebraic system of equations:
\[ \begin{bmatrix} 1 & 0 & 0 & 2 \\ 1 & 0 & 0 & 2 \\ 2 & 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} \tag{1} \]
(a) Determine the general solution.
(b) Determine a basis for the four fundamental subspaces: \( R(A), R(A^T), N(A), N(A^T) \).

Now let \( b = [1 \quad -1 \quad 1]^T \) and consider the vector norm \( \|z\| \overset{\text{def}}{=} \sqrt{z^T z} \).
(c) Determine the set of all possible \( x \) that minimizes the error \( \|b - Ax\|_2 \).
(d) Determine the projection \( P_{R(A)} b \) of the vector \( b \) onto the range of \( A \).
(e) Determine the minimum norm \( x \) which minimizes the error \( \|b - Ax\|_2 \).

Problem 3 (Controllability, State Transfer, Pole Placement)
Consider the LTI system defined by the state space dual \( A = \begin{bmatrix} -p_1 & z - p_2 \\ 0 & -p_2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) where \( z \neq p_1 \) and \( z \neq p_2 \). Sketch a block diagram for the system.
(a) Is the system controllable? Explain your answer. Are there any pole-zero cancellations?
(b) Does there exist a control law which will transfer the state of the system from \( x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) to \( x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) if so, determine a suitable control gain matrix \( G \). If not, explain why.

Problem 4 (Controllability, State Transfer, Pole Placement)
Consider the LTI system defined by the state space dual \( A = \begin{bmatrix} -p_1 & z - p_2 \\ 0 & -p_2 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) where \( z = p_1 \) and \( z \neq p_2 \).
(a) Is the system controllable? Explain your answer. Are there any pole-zero cancellations?
(b) Assuming \( x_o = 0 \), determine an expression for \( x(t) \) using modal analysis concepts. Determine a basis for the set of states that are reachable from \( x_o = 0 \).
(c) Does there exist a control law which will transfer the state of the system from \( x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) to \( x(1) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \) if so, determine one. Also, determine a minimum energy state transferring control law.
(d) Does there exist a control law which will transfer the state of the system from \( x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \) to \( x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \)? If so, determine one. If not, determine what state closest to \( x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \) is reachable. Then determine a control law which will achieve the transfer to this reachable state. Is your control law unique? If not, show how to determine another state transferring control law and determine the minimum energy state transferring control law.
(e) Does there exist a full state feedback control law \( u = -Gx \) such that the closed loop system has poles at \( s = -p_1, -2 \) (where \( p_1 \neq 2 \)) if so, determine a suitable control gain matrix \( G \). If not, explain why.
(f) Does there exist a full state feedback control law \( u = -Gx \) such that the closed loop system has poles at
Problem 5 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple
\[ A = \begin{bmatrix} -p_1 & 1 \\ 0 & -p_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [ z - p_1 \ 1 ] \]
where \( z \neq p_1 \) and \( z \neq p_2 \). Sketch a block diagram for the system.

(a) Is the system observable? Explain your answer. Are there any pole-zero cancellations?
(b) Suppose that \( u = 0 \) and \( y \) is known on \( t \in [0, 1] \). Determine an expression for the set of all possible initial conditions \( x_o \). Can \( x_o \) be determined uniquely? Explain. Determine the minimum norm initial condition.
(c) Can one design an observer with closed loop poles at \( s = -1, -2 \)? If not, explain why. If so, determine a suitable observer gain matrix \( H \). Moreover, determine the state estimation error \( \hat{x} = x - \hat{x} \) when \( u = 0 \), the initial system state is \( x_o = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \), and the initial state estimate (used in your observer) is \( \hat{x}_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \).

Problem 6 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple
\[ A = \begin{bmatrix} -p_1 & 1 \\ 0 & -p_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad C = [ z - p_1 \ 1 ] \]
where \( z \neq p_1 \) and \( z = p_2 \).
(a) Is the system observable? Explain your answer. Are there any pole-zero cancellations?
(b) Suppose that \( u = 0 \) and \( y \) is known on \( t \in [0, 1] \). Determine an expression for the set of all possible initial conditions \( x_o \). Can \( x_o \) be determined uniquely? Explain. Determine the minimum norm initial condition.
(c) Can one design an observer with closed loop poles at \( s = -p_2, -2 \) (where \( p_2 \neq 2 \))? If so, determine a suitable observer gain matrix \( H \). If not, explain why.
(d) Can one design an observer with closed loop poles at \( s = -2, -5 \) (both distinct from \( -p_2 \))? If so, determine a suitable observer gain matrix \( H \). If not, explain why.
(e) Discuss how the answers to (a)-(d) change if \( z = p_1 \) and \( z \neq p_2 \)? Hint: Examine block diagram.

Problem 7 (Model Based Compensator Design)
Consider the linear time invariant plant
\[ P(s) = \frac{2}{s - 5} \]
with state space quadruple \( A_p = 5, \quad B_p = 1, \quad C_p = 2, \quad D_p = 0 \).
(a) Show how to design a model based compensator which satisfies the following design specifications:
   (i) zero steady state error to step reference commands,
   (ii) closed loop poles at \( s = -4 \pm j3, \ s = -100 \pm j100 \).
(b) Discuss how one might minimize the overshoot due to a step reference command.
(c) Summarize the design process if one desires to follow sinusoidal reference commands with frequency \( \omega_o \).