Problem 1 (Analysis of a Continuous-Time Feedback System)

Consider the feedback system defined by the continuous-time LTI plant \( P = \frac{1}{s-1} \), series compensator \( K = \left[ \frac{9(s+30)}{s} \right] \left[ \frac{100}{s+100} \right] \), pre-filter \( W = \left[ \frac{25}{s+5} \right] \), and sensor dynamics \( H = 1 \).

(a) Determine the (approximate) closed loop transfer functions \( T_{ry}, T_{dy} \). What are the closed loop poles, the associated time constant \( \tau \), settling time \( t_s \). What is the system settling time?

(b) What is the (approximate) linear ordinary differential equation relating \( y \) to \( r \) and \( d_i \)?

(c) Approximate the output \( y \) when \( r = 10 + 2 \sin(0.1t - 30^\circ) \) and \( d_i = 2 - \cos(0.1t + 45^\circ) \). Show how to compute all coefficients.

(d) Approximate the steady state output \( y_{ss} \) for the scenario in (c).

(e) Sketch a carefully labeled root locus for the above system. Determine all imaginary crossovers. Clearly label all asymptotes.

(f) Sketch \( |L| \) versus frequency.

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For (a), determine the damping factor \( \zeta \). What is the overshoot to a step command \( M_p \) and the time to peak \( t_p \) for a step command? Plot the response \( y \) to a step \( r \).

For (c), use Simulink to plot \( y \).

For (e), determine the cg for the root locus. Provide a complete stability summary. Compute the upward and downward gain margins \( \uparrow GM, \downarrow GM \).

For (f), provide a complete Bode magnitude and phase plots. Compute the associated phase and delay margins \( PM, DM \). Clearly show \( \uparrow GM, \downarrow GM, PM \) on your plots.

(g) Suppose \( L(j\omega_1) = \frac{1}{1+L}, L(j\omega_2) = \frac{1}{1+L}, L(j\omega_3) = e^{j(M-\omega_0)} \). Let \( S = \frac{1}{1+L} \) and \( T = 1 - S = \frac{T}{1+L} \). Determine \( S(j\omega_1), T(j\omega_1), S(j\omega_2), T(j\omega_2) \). Given the above, determine a lower bound for \( \|S\|_{\infty} \) and \( \|T\|_{\infty} = \max_{\omega} S(j\omega) \) and \( \|T\|_{\infty} = \max_{\omega} T(j\omega) \) ?

Problem 2 (Analysis of a Discrete-Time Feedback System)

Suppose that the above system is discretized with a sampling rate \( \omega_s = 600 \text{ rad/sec} \) using Tustin’s discretization method (trapezoidal rule/bilinear transformation). Consider the feedback system defined by the discrete-time LSI plant \( P_d = 0.005264 \left[ \frac{z+1}{z-1.011} \right] \), series compensator \( K_d = 1.0783 \left[ \frac{z-0.9714}{z-1} \right] \left[ \frac{z+1}{z-0.9714} \right] \), pre-filter \( W_d = 0.01431 \left[ \frac{z+1}{z-0.9714} \right] \), and sensor dynamics \( H_d = 1 \).

(a) Determine the (approximate, “low frequency”) closed loop transfer functions \( T_{ry}, T_{dy} \). What are the approximate closed loop poles? 1% settling time?

(b) What is the (approximate) linear ordinary differential equation relating \( y \) to \( r \) and \( d_i \)?

(c) Approximate the output \( y_n \) when \( r_n = 10 \) and \( d_{i_n} = \cos(0.1nT + 45^\circ) \). Show how to compute all coefficients.

(d) Approximate the steady state output \( y_{ss} \) for the scenario in (c).

(e) Sketch a carefully labeled root locus for the above system. Use all poles and zeros! Determine all unit circle (\( |z| = 1 \)) crossovers.

(f) Sketch \( |L_d| \) versus frequency. Compare with \( |L| \).

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Use MATLAB to compute everything exactly.
Problem 3 (Continuous-Time Design)
Consider the continuous-time LTI plant \( P = \frac{1}{(s-2)(s+6)} \left[ \frac{100}{s+100} \right]^4 \). Design a control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error to step input disturbances \( d_i \), (3) has dominant closed loop poles at \( s = -4 \pm 3j \), (4) attenuates the impact of high frequency sensor noise on the controls, and (5) minimizes the effect of output overshoot when a step reference command is issued. Support your design with a simple root locus plot.

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Use MATLAB to plot Bode plots for \( |L|, \angle L, |S|, |T|, |KS|, |PS|, |Tr_p| \). Show Discuss low frequency command following, low frequency disturbance attenuation, and high frequency noise attenuation. What are the closed loop poles? Plot the response to a step reference command. What is the percent overshoot? Compute all margins.

Problem 4 (Discretization)
Consider the continuous-time LTI system \( L = \left[ g s \left( s + a \right) \right] \left[ cs - \frac{c}{s} \right] \) with \( a = g = 2 \) and \( c = 20 \).
(a) Sketch a root locus. Determine imaginary crossovers. What are the closed loop poles? settling time?
In this problem, you will be discretizing \( L \) using a sampling frequency \( \omega_s = 120 \text{ rad/sec} \) \( (T \approx 0.05) \).
(b) Discretize \( L \) using the bilinear transformation.
(c) Sketch a discrete-time root locus for the \( L_d \) obtained in (b). Determine unit circle crossovers. What are the closed loop poles?
(d) Sketch \( |L(j\omega)| \) and \( |L_d(e^{j\omega T})| \) versus frequency \( \omega \).
(e) Show that \( \lim_{T \to 0} L_d(e^{sT}) = L(s) \) for a fixed \( s \).

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(f) How small can \( \omega_s \) be made before the discrete closed loop system goes unstable?
(g) Use the step invariant method to repeat (b)-(f). Which method can tolerate a smaller \( \omega_s \) before the discrete-time closed loop system goes unstable?

Problem 5 (Discrete Control System Design)
Suppose that we have a discrete-time LSI plant \( P_d = \frac{1}{z-0.5} \). Design a feedback control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error to step output disturbances \( d_o \), (3) exhibits closed loop poles of \( z = \frac{1}{2} e^{j\frac{\pi}{4}} \), (4) attenuates the impact of high frequency sensor noise on the controls (near Nyquist rate), (5) reduces overshoot in output when a step command is issued. Support your design with a simple root locus plot.

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Use MATLAB/Simulink to plot the response to a step reference command. What is overshoot to a step command?

Problem 6 (Control System Design for Overshoot and Settling Time)
Suppose that we have a plant \( P = \frac{1}{s(s+1)} \left[ \frac{100-s}{s+100} \right]^2 \left[ \frac{10^3}{s+10^7} \right]^4 \). Design a feedback control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error to ramp output disturbances \( d_o \), (3) exhibits a settling time of approximately \( t_s = 5 \) sec to step commands, (4) exhibits an approximate overshoot of 9.5% when a step command is issued, (5) attenuates the impact of high frequency sensor noise on the controls. Support your design with a simple root locus plot.

Problem 7 (Control System Design for BW and PM)
(a) Nominal Design. Suppose that we have a plant \( P = P_o = \frac{1}{s^2(s-1.5)(s+3)^2} \left[ \frac{6}{s+2} \right]^2 \left[ \frac{30-s}{s+30} \right]^4 \left[ \frac{10^3}{s+10^7} \right]^2 \). Design a feedback control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error
to ramp input disturbances $d_t$, (3) exhibits a unity gain crossover of $\omega_g = 3 \text{ rad/sec}$, (4) exhibits a phase margin $PM = 60^\circ$. Support your design with a simple root locus plot, (5) high frequency sensor noise has little impact on the controls, (6) overshoot to a step reference command is minimized.

(b) Robustness. Now suppose that $P = P_o \left[ \frac{0.5\Omega_s}{s+0.5\Omega_s} \right] e^{-s\tau} \Omega_s = k\omega_g$ and $k > 0$. Assuming that the control system designed in (a) is used with $P$, how small can $k$ be reduced before the phase margin is reduced to $30^\circ$?

(c) New Design. Design a new control system that guarantees the original design specifications for the new plant $P$ with the $k$ determined in (b).

EXTRA CREDIT: Suppose we also wanted zero steady state error to a sinusoidal input disturbances $d_i$? How would you alter your design? Explain.

Problem 7 (Analysis of a Discrete-Time Feedback Control System)
Consider a plant $P = \frac{1}{z}$ and controller $K = \frac{z}{z-1}$ within a classic negative feedback configuration. (a) Compute $T_{ry}$. (b) Determine the closed loop poles. Discuss the stability of the closed loop system. (c) Determine and explain the dc gain for $T_{ry}$. (d) Determine $y$ when $r$ is a unit step. (e) Determine $r$ such that $y_{ss} = 1 + \sin\left(\frac{\pi}{2}n - 45^\circ\right)$ when $d_i = \sin(0.01n + 30^\circ)$. (f) Plot a root locus for $L = PK$. Determine break points and unit circle crossovers.