Problem 1 (Analysis of a Feedback System, Root Locus)
Consider the feedback system defined by the open loop transfer function

\[ L \equiv PK = 5.56 \left[ \frac{s^2 + 1.5616s + 1.0004}{s^3} \right] \left[ \frac{10^4}{s^2 + 2(0.7)(100)s + 10^4} \right] \left[ \frac{s^2 + 2(0.7)(10^4)s + 10^8}{10^8} \right] \left[ \frac{10^{12}}{s^2 + 2(0.7)(10^6)s + 10^{12}} \right] \left[ \frac{10^{10} - s}{10^8 + s} \right] \left[ \frac{10^{10}}{s + 10^{10}} \right]^5. \]

(a) Use the angle criterion to show that \(-1 + j0.75\) and \(-3.56\) are approximate closed loop poles.
(b) What is the associated (approximate) time constant \(\tau\), settling time \(t_s\), undamped natural frequency \(\omega_n\), damping factor \(\zeta\), damped natural frequency \(\omega_d\), overshoot parameter \(M_p\) and time to peak parameter \(t_p\)?
(c) Give an expression for the approximate response \(y\) to a unit step reference command.
(d) How will actual overshoot compare with \(M_p\). How can this be improved?
(e) Approximate the steady state output \(y_{ss}\) when \(r = 10 + \sin(0.1t - 30^\circ)\) and \(d_0 = \cos(10t + 45^\circ)\).
(f) Construct a root locus for the negative feedback system defined by \(L\) above. Determine the number of asymptotes, the angle of each asymptote, the center of gravity, all imaginary crossovers. Carefully label all important features on your plot. Determine the upward gain margin \(\uparrow GM\) and downward gain margin \(\downarrow GM\).

Problem 2 (Bode) Sketch Bode magnitude and phase plots for the open loop transfer function \(L\) given above. Identify all phase crossover frequencies on your plot. How do these relate to your root locus plot? Identify the upward gain margin \(\uparrow GM\), downward gain margin \(\downarrow GM\), phase margin \(PM\), and associated frequencies on your plots. Compute the phase margin \(PM\) and delay margin \(DM\).

Problem 3 (Nyquist) Sketch a Nyquist plot for the open loop transfer function \(L\) given above. Identify all phase crossover frequencies on your plot. How do these relate to your root locus plot? Identify the upward gain margin \(\uparrow GM\), downward gain margin \(\downarrow GM\), and phase margin \(PM\) on your plot. Provide a complete stability summary that accounts for all closed loop poles for each \(k\) range.

Problem 4 (Design for Overshoot and Settling Time) Suppose that we have a plant

\[ P = \frac{1}{s(s-1)(s+2)(s+3)} \left[ \frac{100}{s + 100} \right]^2 \left[ \frac{100 - s}{100 + s} \right]. \]

Design a feedback control system such that the design is as simple as possible and the closed loop system (1) is stable, (2) exhibits zero steady state error to step commands \(r\), (3) exhibits an approximate settling time of \(t_s = 2\) sec to step commands \(r\), (4) exhibits an approximate overshoot of 9.5% to step reference commands, (5) attenuates impact of high frequency sensor noise on the controls. Describe your design process. Support your design with a simple root locus plot.

Problem 5 (Design for Pole Placement) Suppose that we have a plant

\[ P = \frac{1}{(s-1)(s+2)(s+3)} \left[ \frac{100}{s^2 + 2(0.1)(10)s + 100} \right] \left[ \frac{z - s}{z + s} \right]^2\]

with \(z = 10\). Design the simplest feedback control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error to step commands \(r\), (3) exhibits an approximate settling time of \(t_s = 7.5\) sec to step commands \(r\), (4) exhibits approximate overshoot of 1.5% to step reference commands, (5) attenuates impact of high frequency sensor noise on the controls. Describe your design process. Do you need a pre-filter? Comment on the flaws (if any) with your approach. Support your design with a root locus plot. How small can \(z\) be made before your design structure must be changed? Specify the new structure.

Problem 6 (Design for BW, PM) Suppose that we have a plant \(P = \frac{e^{-\Delta}}{s(s-1)(s+3)(s+6)}\) with \(\Delta = 1\) sec. Design a feedback control system (that is as simple as possible) such that the closed loop system (1) is stable,
(2) exhibits zero steady state error to ramp input disturbances $d$, (3) exhibits a unity gain crossover frequency of $\omega_g = 3$ rad/sec, (4) exhibits a phase margin of $PM = 60^\circ$, (5) attenuates impact of high frequency sensor noise on the controls. Describe your design process. Support your design with a rough Bode phase plot. How would you reduce overshoot due to step reference commands? Approximately how large a $\Delta$ can be tolerated by your design structure? How would you modify your design for $\Delta$ values slightly larger than the above critical value?