Impulse, Step, and Frequency Response of a Single Stage Common Emitter Audio Amplifier

Here is an application of signals & systems to a typical EE problem: The analysis and design of a (low frequency) audio amplifier. Of course, the analysis to presented can also be applied to (high frequency) optical amplifiers.

Consider the following Common Emitter audio amplifier:

![Amplifier Circuit Diagram]

Data:
- $h_{fe} = 200$
- $h_{ie} = 1 \text{K} \Omega$
- $R_e = 0.1 \text{K} \Omega$
- $R_b = 0.6 \text{K} \Omega$
- $C_e = 1000 \mu \text{F}$

In this problem, the coupling capacitors $C_{c1}$ and $C_{c2}$ in the input and collector circuits will be neglected ($C_{c1}=C_{c2}=\infty$) in order to focus on the emitter bypass capacitor $C_e$.

A small signal model for the above amplifier is as follows:

![Small Signal Model Diagram]

Using KVL and KCL, one can show that the transfer function from $i_i$ to $i_e$ is given by:

$$\frac{i_e}{i_i} = H(s) = \frac{R}{\frac{s+a}{s+b}}$$

where $R$, $a$, and $b$ are found as follows.
\[ R_{be} = \frac{R_b + \frac{h_i e}{h_f e + 1}}{h_f e + 1} = \frac{0.6 + 1}{200 + 1} = 0.00796 \, \text{k}\Omega = 7.96 \, \Omega \]

\[ R = \frac{R_b}{R_{be}} = \frac{0.6 \, \text{k}}{0.00796 \, \text{k}} = 75.38 \, \text{(37.5 dB)} \leq \text{Midband Current Gain} \]

\[ a = \frac{1}{R_{ce}} = \frac{1}{(0.1 \, \text{k}A)(1000 \, \mu F)} = \frac{1}{100 \, \text{msec}} = 0.01 \, \text{krad/sec} \]
\[ = 10 \, \text{rad/sec} \]
\[ \frac{a}{2\pi} = 1.59 \, \text{Hz} \]

\[ R_T = \frac{R_e}{R_{be}} = \frac{R_e}{R_{be} + R_{be}} = \frac{(0.1)(0.00796)}{0.1 + 0.00796} = 0.00737 \, \text{k}\Omega \]
\[ = 7.37 \, \Omega \]

\[ b = \frac{1}{R_T \times R_e} = \frac{1}{(0.00737)(1000)} = 0.13 \, \text{krad/sec} = 135.7 \, \text{rad/sec} \]
\[ \frac{b}{2\pi} = 21.6 \, \text{Hz} \]

\[ H(\omega) = R \frac{a}{b} = \frac{(75.38)(0.01)}{(0.13)} = 5.8 \, \text{(15.27 dB)} \leq \text{dc gain} \]

**Visualization of Amplifier Frequency Response (Bode Asymptotes)**

- 20log|H(j\omega)| dB
- 75.38 (37.5 dB)
- 5.8 (15.27 dB)
- \(\alpha = 1.59 \, \text{Hz}\) (\(a = 10 \, \text{rad/sec}\))
- \(b = 21.6 \, \text{Hz}\) (\(b = 135.7 \, \text{rad/sec}\))

Determined by transistor Cbe, Cbc caps; must roll-off due to other circuit capacitance. Not shown in circuit.
a) Compute the impulse response of our audio amplifier

Use

$$H(s) = k \frac{s+a}{s+b} \left[ \frac{c}{s+c} \right]$$

as the transfer function. Here $c$ is some large positive real number. ($c \gg b > a$) Assume zero initial conditions.

solution:

$$H(s) = \frac{A}{s+b} + \frac{B}{s+c}$$

$$h(t) = [A e^{-bt} + B e^{-ct}] u(t)$$

$$A = \lim_{s \to -b} (s+b) H(s) = \lim_{s \to -b} k \frac{(s+a)c}{s+c} = k c \left[ \frac{a-b}{c-b} \right] \approx -k (b-a)$$

$$B = \lim_{s \to -c} (s+c) H(s) = \lim_{s \to -c} k \frac{s+a}{s+b} c = k c \left[ \frac{a-c}{b-c} \right] \approx k c$$

$$h(t) \approx k \left[ c e^{-c t} - (b-a) e^{-b t} \right] u(t)$$

$$\approx k \delta(t) - (b-a) e^{-b t} u(t)$$

$$\approx 75.38 \delta(t) - (135.7 - 10) e^{-135.7 t} u(t)$$

$$\approx 75.38 \delta(t) - 125.7 e^{-135.7 t} u(t)$$
b) Compute the step response of our audio amplifier

\[ H(s) = K \left[ \frac{s + a}{s + b} \right] \frac{c_1}{s + c_1}, \]

Assume zero initial conditions. Use \( c_1 = 5 \times 10^4 \) rad/sec (7958 Hz).

\[ A = \lim_{s \to 0} s I_e(s) = \lim_{s \to 0} H(s) = H(0) = \text{dc gain} = 5.8 \]

\[ B = \lim_{s \to -b} (s + b) I_e(s) = \lim_{s \to -b} K \left[ \frac{s + a}{s + b} \right] \frac{c_1}{s + c_1} = k \frac{(a-b)}{b} \frac{c_1}{c_1-b} \]

\[ \approx k \left(1 - \frac{a}{b}\right) \]

\[ \approx 75.38 \left(1 - \frac{10}{135.7}\right) \]

\[ \approx 69.83 \]

\[ C = \lim_{s \to -c_1} (s + c_1) I_e(s) = \lim_{s \to -c_1} K \left[ \frac{s + a}{s + b} \right] \frac{c_1}{s + c_1} = -k \frac{a-c_1}{b-c_1} \approx -k \]

\[ \approx -75.38 \]

\[ i_e(t) = \left[ 5.8 + 69.83 e^{-135.7t} - 75.38 e^{-5 \times 10^4 t} \right] u(t) \]