Example: Additive Matrix Perturbations

from Ref 911022 part C

If |\(\Delta| < 5\) then, \(5 + \Delta\) is nonsingular (invertible)

U

This is a sufficient condition for U

Is our condition necessary? NO

counter example:

\[
\Delta = -20
\]

\[
|\Delta| > 5 \quad 5 + \Delta = 5 - 20 = -15
\]

When is our sufficient condition a necessary condition?

Suppose \(\Delta\) is any number whose sign we don't know.

\[
5 + \Delta \text{ is nonsingular} \iff |\Delta| < 5
\]

nec.

suff.

for any of the above admissible \(\Delta\)'s

Assume

(1) \(5 + \Delta\) is nonsingular for any admissible \(\Delta\)

(2) \(|\Delta| > 5\)

This is a proof by contradiction

counter example:

\[
\Delta = -5 \quad \text{satisfies (2)}
\]

but \(5 + \Delta = 5 - 5 = 0\) is singular,

" contradicts (1)

It must therefore be that \(|\Delta| < 5\)! QED
Modal Analysis

Claim: If \( x_0 = k \) then, \( x(t) = e^{\lambda t} x_k \)

Good for \( \lambda_k \) real

Issue: What if \( \lambda_k \) and \( x_k \) are complex?

Example:
\[ A = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix} \]

\[ \det(\lambda I - A) = \det \begin{bmatrix} \lambda & -1 \\ -2 & \lambda + 2 \end{bmatrix} = \lambda^2 + 2\lambda + 2 \]

\[ \lambda_{1,2} = -1 \pm j1 \]

Recall:
\[ x_i = \begin{bmatrix} 1 \\ \lambda_i \end{bmatrix} \Rightarrow x_i = \begin{bmatrix} 1 \\ -1+j1 \end{bmatrix}, \quad x_L = \begin{bmatrix} 1 \\ -1-j1 \end{bmatrix} \]

- How do we excite the \( e^{\xi \cos \omega t} \) mode associated with this system?
- How do we pick \( x_0 \)?

Example [slightly modified]

\[ (s^2 + 2s + 2)(s+1) = s^3 + s^2 + 2s + 2 \]

\[ + 2s + 2 \]

\[ s^3 + 3s^2 + 4s + 2 \]

\[ x = Ax \quad A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2 & -4 & -3 \end{bmatrix} \text{-- companion form matrix} \]
show that \( \det(\lambda I - A) = \lambda^3 + 3\lambda^2 + 4\lambda + 2 \)
\[= (\lambda^2 + 2\lambda + 2)(\lambda + 1) \]

Show that \( x_i = \begin{bmatrix} 1 \\ \lambda_i \\ \bar{\lambda}_i \end{bmatrix} \)

\[x_1 = \begin{bmatrix} 1 \\ -1+j1 \\ (-1+j1)^2 \end{bmatrix} \quad x_2 = \begin{bmatrix} 1 \\ -1-j1 \\ (-1-j1)^2 \end{bmatrix} \quad x_3 = \begin{bmatrix} 1 \\ -1 \\ (-1)^2 \end{bmatrix} \]

How do we choose \( x_0 \) s.t. \( x(t) \) exhibits the expected \( e^{-t}\cos t \) mode and not the \( e^t \) mode.

**Question**

Suppose we have \( \dot{x} = AX \), \( x(0^-) = x_0 \)

and \( A \) has complex eigenvalues.

\( \lambda_k = \sigma_k + j\omega_k \quad \bar{\lambda}_k = \sigma_k - j\omega_k \)

and associated right eigenvectors

\( x_k = y_k + jz_k \quad \bar{x}_k = y_k - jz_k \)

How do we excite the expected \( e^{\sigma_k t}\cos (\omega_k t) \) mode by itself without exciting any of the other modes.

Choose any linear combination

\( x_0 = \alpha y_k + \beta z_k \)