1. **Uncertainty.** Be familiar with four different uncertainty characterizations. Know relationships between different uncertainty characterizations; e.g. Given SISO $P$ and $P_a$ you should be able to (1) determine $\Delta_a$, $\Delta_m$, $\Delta_d$, $\Delta_f$, (2) relate $\Delta_a$, $\Delta_m$, $\Delta_d$, $\Delta_f$.

2. **Small Gain Theorem.** You should be able to (1) precisely state the small gain theorem. (2) explain the sufficiency part of small gain condition; i.e. why is the condition sufficient? how would one prove that it is sufficient (no proof is needed...just the idea like we did in class), (3) state when the small condition is necessary, (4) prove the necessity part (proof via contradiction and using a counterexample as we did in class). Assuming that the two subsystems $T$ and $\Delta$ are stable: What can you say if the small gain condition is passed? (This is easy case.) What can you say if the small gain condition is violated? (This requires more thought. This is the heart of the contradiction part of the proof. Explain why counterexample is relevant here.)

3. **Forming Generalized Plant.** Consider a nominal plant $P = \frac{s}{s+1}$ with state $x_p$ with weightings $W_1 = k_1 \frac{s+1}{s+1}$ with state $x_1$ and state space $A_1 = -p_1$, $B_2 = z_1 + p_1$, $C_1 = D_1 = k_1$, $W_2 = k_2$, $W_2 = k_3(s + z_3)$. Sketch a block diagram of the overall closed loop system. Determine the transfer function matrix for the generalized plant. Let $x = [x_p, x_1]^T$ represent the state of the closed loop system. Determine the state space representation for the generalized plant.

4. **Weighted $H^\infty$ Mixed-Sensitivity Minimization.** Given $P = \frac{s}{s+1}$, use computer to determine weightings $W_1, W_2, W_3$ which yield a controller that approximates $K = 9 \left[ \frac{s+1}{s} \right]$. Note that dominant closed loop poles are at $s = -4 \pm j3$. Describe the design procedure. How did you select the weightings? Come in with design procedure, weightings, and minimum $\gamma$ on your sheet!

5. **State Space Arithmetic.** Forming state space representations for closed loop systems; e.g. LQR servo, classical feedback structure.

6. **Block Diagram Interconnections.** Consider a classic SISO negative feedback system with plant $P = \frac{1}{s+1}$ and controller $K = 1.1$ subject to an input step disturbance $d_i$. Discuss why the following blows up in MATLAB:

```matlab
t = 0:.001:35;  % You may need a final time greater than 35 on your computer. Why?
plant = tf(1,[1 -1]);
comp = 1.1;
sen = 1/(1 + plant*comp);
step( plant*sen,t)
grid on
```

Specify $a, b, c, d$ such that the following yields the correct response to the step disturbance $d_i$:

```matlab
t = 0:.001:50;
step( ss(a,b,c,d),t)
grid on
```

7. **Poles and Transmission Zeros.** Given an LTI system, you should be able to determine poles, transmission zeros, and associated transmission zero directions. More specifically, for the classical LTI feedback structure, how would you determine transmission zeros of the closed loop transfer function matrix from $r$ to $u$? $d_i$ to $y$? Hint: Just need relevant closed loop matrices. Is there a simpler way? Hint: zeros of $K$, zeros of $P$. 

8. **Push-Pop Effect.** Explain push-pop tradeoffs for the following unstable non-minimum phase plant $P = \frac{z^4}{z^2 + 1}$ for different values of $z$; e.g. $z = 100, 20, 5$.

Fact: Hypersonic vehicles are know to possess such a structure in the elevator to flight path transfer function. All elevator controlled airplanes/missiles have a right half plane zero in their elevator to flight path transfer function.

9. **Symmetric Matrices.** Symmetric positive definite and positive-semi-definite matrices have a wide presence throughout the multivariable control literature; e.g. LQR, Kalman Filtering, LQG. (a) Give an example of a simple $2 \times 2$ symmetric positive definite matrix. In general, what are ways to test for symmetric positive definiteness? (b) Give an example of a simple $2 \times 2$ symmetric positive semi-definite matrix. In general, what are ways to test for symmetric positive semi-definiteness?

10. **Stabilizability and Detectability.** Consider the series cascade of the following three systems: $S_1$ followed by $S_2$ followed by $S_3$. Their respective state space descriptions are as follows:

$A_1 = -1, B_1 = -(z_1 + 1), C_1 = 1, D_1 = 1$, state $x_1$, input $u_1$, output $y_1$

$A_2 = 1, B_2 = C_2 = 1, D_2 = 0$, state $x_2$, input $y_1$, output $y_2$

$A_3 = -1, B_3 = -(z_3 + 1), C_3 = 1, D_3 = 1$, state $x_3$, input $y_2$, output $y_3$

(a) Determine transfer function for each system.

(b) Determine state space representation for cascade system $S_3S_2S_1$ ($S_1$ at input, $S_3$ at output).

(c) Determine conditions for the system to be stabilizable.

(d) Determine conditions for the system to be detectable.

Hint for (c) and (d): Think in terms of pole-zero cancellations.

11. **LQR and Kalman Filter.** You should know the basic LQR problem statement, assumptions, solution, and properties. You should be able to do computations for 1st and 2nd order systems. Be prepared to set up equations for Riccati matrix entries (not solve). Do “exploring LQR problem.”

Same for Kalman filter. You should be able to design a simple observer - specify all equations, assumptions, and compute filter gain matrix; e.g. $P = \frac{1}{s-1}$ or $P = \frac{1}{s^2}$. Make simplifying assumptions about uncertainty sources; e.g. unit intensities for process and measurement noise.

12. **LQR Servo.** You should be able to do LQR servo designs for $P = \frac{1}{s-1}$ with and without integral action. Same for $P = \frac{100}{s-1}$. Be prepared to set up equations for Riccati matrix entries (not solve). Describe procedure. Sketch block diagram. Give formula for $G$ and $G_L$.

13. **LQR and Kalman Filter Robustness Properties.** What are the LQR robustness properties? Where do they come from? Recall LQ-FDE. Illustrate $S$ and $T$ properties for SISO systems via Bode. Illustrate $S$ properties via Nyquist. Sketch how a system can have wonderful gain and phase margins but terrible robustness. Hint: Number of encirclements of -1 point! Same for Kalman filter.

14. **LQR and Kalman Filter Loop Shaping.** Describe procedure for LQR loop shaping; i.e. matching singular values at low, high, all frequencies. Same for Kalman filter.

15. **SVD.** You should be able to interpret SVD for transfer function matrix at specific frequency. What is worse-case input and resulting steady state output? (We did this for PUMA.) Discuss how units change the singular values.

16. **LQG and LQG/LTR.** State LQG problem statement, assumptions, solution, and properties. Describe LQG/LTR procedure at output and at input. Specify sufficient conditions for LTR (minimum phase condition, special limiting conditions). Provide block diagram proof of LTRO and LTRI. Describe how we can have LTR when plant is non-minimum phase.

17. **Pre-Filter Design.** Consider the open loop transfer function $L = PK = \frac{1}{H}$ ($H = 1$). (a) Compute complementary sensitivity $T$. (b) Determine closed loop poles. (c) Is closed loop system stable, marginally stable, unstable? (d) What is the response to a unit step reference command? (Must do this analytically!) (e) What is overshoot? (f) Now consider $T_y = WT$ where $W = \frac{1}{1+z}$. Select $z$ such that the resulting steady state peak output excursions from unity are at most 0.05. (g) Does the above pre-filter fix the design? Carefully explain! Explain why the design is still bad. Give very specific reasons.

18. **Linearization.** Explain how a carefully constructed linear model can be used for nonlinear fishery system from first exam. Justify your answer.