Problem 1 (Control System Design)

(a) **Design.** Suppose that we have a plant \( P = \frac{1}{s-2} \left[ \frac{500-s}{s+500} \right]^5 \). Design a feedback control system such that the closed loop system (1) is stable, (2) exhibits zero steady state error to step \( r \), (3) exhibits a settling time to step commands \( r \) of \( t_s \approx 20 \), (4) overshoot close to 17\% when a step command is issued. (5) impact of high frequency noise on control is minimized.

(b) **Bode.** Sketch Bode magnitude and phase plots for the open loop transfer function \( L \) above. Identify the upward gain margin \( \text{Gm} \), downward gain margin \( \text{DGm} \), phase margin \( \text{PM} \), and associated frequencies on your plots. Compute all gain, phase and delay margins and the associated frequencies.

(c) **Root Locus.** Sketch a root locus. Compute all imaginary crossovers and angles of asymptotes.

(d) **Bounds for \( S \) and \( T \).** Given the margins for your design, determine lower bounds for the peak sensitivity \( S \) and complementary sensitivity \( T \).

(e) **Bounds for Margins.** Suppose the peak sensitivity \( S \) and complementary sensitivity \( T \) are bounded above by \( \alpha \) and \( \beta \) (both \( \geq 1 \)), respectively. What can be said about the associated gain and phase margins? What if \( \alpha = \beta = 1 \)?

Problem 2 (Model Based Compensator Design)

Consider a plant \( P = \left[ \frac{s-z}{s-p} \right] = [A_p, B_p, C_p, D_p] = [p, z - p, 1, -1] \) with \( z = 10 \) and \( p = 2 \). (a) Show how to design a (model-based) control system such that the closed loop system (i) is stable, (ii) exhibits zero steady state error to step output disturbances, (iii) exhibits a settling time of approximately 10 sec, (iv) exhibits an overshoot of approximately 10\%, (v) attenuates the impact of high frequency sensor noise on the controls.

Support your design with a rough root locus plot. Just show how to set up the problem. You need not solve the equations. (b) Let \( (A_k, B_k, C_k) \) denote the state space representation for the final controller \( K \). Determine the state space representation for the closed loop system from \( (r, d, d_a, n) \) to \( (x_p, x_k) \), \( (y, u) \).

Problem 3 (LQ Properties) For each case explain you answer. Provide \( (A, B, M, \rho, G) \) when applicable.

(a) Can \( L = \frac{40}{s-2} \) be an LQ loop? (b) Can \( L = \frac{3}{s-3} \) be an LQ loop? (c) Can \( L = \frac{5}{s-2} \) be an LQ loop? (d) Can \( L = \frac{s}{s-3} \) be an LQ loop? (e) Can \( L = \frac{s+1}{s^2} \) be an LQ loop? (f) Can \( L = \frac{1}{s+1} \left[ \frac{1000-s}{s+1000} \right] \) be an LQ loop?

(g) Can \( L = \frac{5(s+1)}{s(s-1)} \) be an LQ loop? (h) Can \( L = \frac{2(s+1)}{s(s-1)} \) be an LQ loop?

Problem 4 (LQ Servo)

Consider \( P_o = \left[ \frac{s-z}{s-p} \right] \). (a) Show how to design an LQ servo that will guarantee zero steady state error to step reference commands. Clearly explain the process. Sketch a relevant LQ servo block diagram. Please specify the Riccati gain matrix \( K_p \) and the control gain matrix \( G_p \). Hint: \( J = \int_0^\infty (y^2 + pu^2) \) dt, \( y = Mx \). Now suppose \( a = 2, z = 20, \rho = 1 \). (b) Determine \( G_o \) for \( \rho = 1 \). Compute \( G_{LQ} \) and the associated phase margin \( \text{PM} \). (c) Plot a root square locus for your \( G_{OL} \). (d) Determine \( K_p \) and \( G_p \) for \( \rho \to 0, \rho \to \infty \). Explain each!

Problem 5 (Small Gain Theorem)

Consider a negative feedback control system with nominal plant \( P_o = \frac{1}{s} \) and controller \( K = 2 \left[ \frac{s-z}{s-p} \right] \). Consider a truth plant \( P = P_o \left[ \frac{1600}{s^2 + 80\zeta s + 1600} \right] \) with \( \zeta > 0 \). (a) What is the minimum damping factor \( \zeta \) for the closed loop system to be on the verge of instability? (b) What do the 4 associated (additive, multiplicative, divisive, feedback) small gain theorem tests tell us? Please show all relevant Bode plots and discuss what each test implies.

Problem 6 (Loop Transfer Recovery at the Plant Input)

(a) Give a sufficient condition on the plant \( (A, B, C) \) for LTRI. (b) Prove LTRI.