EEE582 Exam 2, Fall 2016 — A.A. Rodriguez
Rules: One 8.5 × 11 sheet permitted, calculators permitted, open minds. — GWC 352, 965-3712

Problem 1 (Gaussian Elimination, Fundamental Subspaces, General Solution, Projections)
Consider the system of linear algebraic equations: $Ax = b$ with $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$.

(a) Parameterize the set of all solutions x.

(b) Determine a basis for each of the four (4) fundamental subspaces: $\mathcal{R}(A)$, $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$.

(c) Given that b does not lie within $\mathcal{R}(A)$, determine the set of all x which minimizes the Euclidean norm (distance) $\|b - Ax\|$. Amongst these, how would you determine the minimum norm solution? ■

Problem 2 (Controllability, Observability, Modal Analysis, MBC)
Consider the LTI system defined by the state space quad

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 1 & -1 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, D = 0.$$

(a) Sketch a block diagram for the system.
(b) Is the system controllable? Explain your answer using multiple tests. Which modes are controllable? uncontrollable? Explain!
(c) Determine a symbolic expression for $x(t)$ when $x(0) = 0$.
(d) Determine the set of all states that are reachable from $x(0) = 0$?
(e) Is $x_f = x(1) = [\ 0\ 2\ 0\ ]^T$ reachable from $x(0) = 0$? If so, construct a minimum energy control law. If not, find the closest reachable state and a suitable minimum energy state transferring control law. Is $x_f = x(1) = [\ 1\ -1\ 0\ ]^T$ reachable from $x(0) = 0$? If so, construct a minimum energy control law. If not, find the closest reachable state and a suitable minimum energy state transferring control law.
(f) Specify the form of $det(sI - A + BG)$. Can $A - BG$ possess the eigenvalues 5, -2, 5? Explain.

(g) Is the system observable? Explain your answer using multiple tests. Which modes are observable? unobservable? Explain!
(h) Determine a symbolic expression for y when $u(t) = 0$ and $x(0) = x_o$.
(i) Specify the form of $det(sI - A + HC)$. Can $A - HC$ possess the eigenvalues 5, -2, 5? Explain.
(j) Show 2 methods for determining the set of all possible initial conditions x_o when $u = 0$.

(k) **Modal Analysis.** Show how to choose x_o in order to separately excite each mode of the above system.

(l) **MBC.** Suppose we wish to design a model based compensator (MBC) such that the closed loop system exhibits zero steady state error to step commands and a settling time less than 5 sec. Can this be done? Explain. ■

Problem 3 (Transmission Zeros) Consider the linear time invariant (LTI) system: $\dot{x} = Ax + Bu, y = Cx + Du$ with $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}$, $D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. (a)Sketch a block diagram for the system. (b) Determine the system transfer function matrix. (c) Determine the associated system poles. (d) Determine the associated system transmission zeros and the associated input and state directions. (e) Clearly explain the significance of the above. ■

Problem 4 (EXTRA: State Space Realization) Provide an observability canonical form state space realization for the LTI system $H(s) = \left[\frac{s^2+s+1}{s^2}\right]$. Sketch a block diagram. Indicate all state variables on your diagram. Clearly discuss the controllability and observability properties. ■