EEE582 Exam 1, Fall 2016 A.A. Rodriguez
Rules: One 8.5 × 11 sheet permitted, calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Gaussian Elimination, Fundamental Subspaces, General Solution, Projections) Consider the system of linear algebraic equations: $Ax = b$

with $A = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 8 & 10 & 12 \\ 3 & 6 & 10 & 12 & 15 & 18 \\ 5 & 10 & 16 & 20 & 25 & 30 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$. (a) Parameterize the set of all solutions x.

(b) Determine a basis for each of the four (4) fundamental subspaces: $\mathcal{R}(A)$, $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$.

(c) Assume $b \in \mathcal{R}(A)$. Determine a solution that lies within $\mathcal{R}(A^T)$. What can be said about such a solution?

(d) Given that b does not lie within $\mathcal{R}(A)$, determine the set of all x which minimizes the Euclidean norm (distance) $\|b - Ax\|$. Amongst these, how would you determine the minimum norm solution? ■

Problem 2 (Controllability and Observability) Consider the LTI system defined by the state space representation: $\dot{x} = Ax + Bu, y = Cx$ with $A = \begin{bmatrix} 1 & 0 & -1 \\ 3 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 3 & 2 & -1 \end{bmatrix}$, $D = 1$.

(a) Sketch a block diagram for the system.
(b) Is the system controllable? Explain your answer using multiple tests. Which modes are controllable? uncontrollable? Explain!
(c) Determine a symbolic expression for $x(t)$ when $x(0) = 0$.
(d) Determine the set of all states that are reachable from $x(0) = 0$?
(e) Determine the form of $det(sI - A + BG)$. Can $A - BG$ possess the eigenvalues 5, 5, 5? Explain.
(f) Is the system observable? Explain your answer using multiple tests. Which modes are observable? unobservable? Explain!
(g) Determine a symbolic expression for y when $u(t) = 0$ and $x(0) = x_o$.
(h) Determine the form of $det(sI - A + HC)$. Can $A - HC$ possess the eigenvalues 5, 5, 5? Explain.
HOME: (i) Is $x(1) = [\ 0.3162 \quad -0.9487 \quad 0\ ]^T$ reachable from $x(0) = 0$? Explain. If so, show how to construct a minimum energy state transferring control law. If not, determine the closest reachable state and then show how to construct a minimum energy state transferring control law.
Is $x(1) = [\ 0.3162 \ \ 0.0513 \ \ 0\ ]^T$ reachable from $x(0) = 0$? Explain. If so, show how to construct a minimum energy state transferring control law. If not, determine the closest reachable state and then show how to construct a minimum energy state transferring control law.
(j) What closed loop poles can be achieved with full state feedback? Explain and then prove your claim.
(k) Determine the set of all possible initial conditions x_o. Show several methods for determining x_o. Determine the minimum norm x_o.
(l) Show how you would construct an observer for the above system. Provide a detailed block diagram. What closed loop poles can be achieved via H for $(A - HC)$? Explain and then prove your claim. ■

Problem 3 (Modal Analysis) Consider the linear dynamical system: $\dot{x} = Ax, x(0) = x_o$ with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -25 & -6 \end{bmatrix}$. (a) Determine the system's characteristic equation? (b) Determine all eigenvalues of A. (c) Determine a set of linearly independent eigenvectors for A. Is A diagonalizable? (d) Compute x. (e) Determine how each mode can be independently excited via carefully selected (real) initial conditions. Determine the (real) response for each initial condition. **HOME:** Repeat above for $\dot{x} = A^T x, x(0) = x_o$. ■

Problem 4 (State Space Realization and Arithmetic) Determine a state space representation for the feedback system with external signals (r, d_i, d_o, n), state $x = [\ x_p \quad x_k\ ]^T$, and outputs (y, u, e) defined by the following equations:

$$
\begin{aligned}
e &= r - y - n & u_p &= u + d_i \quad (1)\\
\dot{x}_k &= A_k x_k + B_k e & \dot{x}_p &= A_p x_p + B_p u_p \quad (2)\\
u &= C_k x_k + D_k e & y &= C_p x_p + D_p u_p + d_o. \quad (3)
\end{aligned}
$$

with $D_p = D_k = 0$. (b) Now suppose $A_p = -1$, $B_p = C_p = 1$, $A_k = 0$, $B_k = C_k = 1$. Determine $T_{d_i u}$ and the associated step response. ■

HOME: (c) Now assume that $M_o \stackrel{\text{def}}{=} (I + D_p D_k)^{-1}$ and $M_i \stackrel{\text{def}}{=} (I + D_k D_p)^{-1}$ are well defined. Determine a symbolic closed loop state space representation.

(d) Provide four distinct state space realizations for the following LTI system $H(s) = \left[\frac{6s^2+12s}{3(s+1)(s^2+s+1)}\right] + 3$. Sketch a block diagram for each realization. Indicate all state variables on your diagrams. Clearly discuss the controllability and observability for each representation.

Problem 5 (State Computation) Consider the linear system $\dot{x} = Ax + Bu, y = Cx$ with

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & 2 & 10 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) Sketch a block diagram for the system. (b) Compute x and y when $u = \delta(t)$ (unit impulse) and $x(0) = 0$. ■

Problem 6 (Transmission Zeros) Consider the linear time invariant (LTI) system: $\dot{x} = Ax + Bu, y = Cx + Du$ with $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 0 \\ -94 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$. (a)Sketch a block diagram for the system. (b) Determine the system transfer function matrix. (c) Determine the associated system poles. (d) Determine the associated system transmission zeros and the associated input and state directions. (e) Use the system block diagram to illustrate the significance of the associated input and state directions. Show how information is propagated through the system. ■

Problem 6 (Model Based Compensator) Consider the linear time invariant (LTI) plant $P(s) = \frac{z-s}{s-p}$ with $z, p > 0$, $A_p = p$, $B_p = z - p$, $C_p = 1$, $D_p = -1$ and variables (u_p, x_p, y_p). Let $p = 1$ and $z = 10$.
(a) Design a model based compensator which satisfies the following design specifications: (i) zero steady state error to step reference commands, (ii) closed loop poles at $s = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$, $s = -10 \pm j10$.
Show how you would determine the final compensator K. (b) Determine the final compensator K.
Repeat the above for $z = 5, 2$. For each of these systems (and the one above), do the following.
(c) Use MATLAB to verify that your design achieves the desired closed loop poles. (d) Let $L \stackrel{\text{def}}{=} PK$ denote the open loop transfer function. Use MATLAB to sketch Bode magnitude and phase plots for L. (e) Let $S = \frac{1}{1+L}$, $T = 1 - S = \frac{L}{1+L}$. Use MATLAB to plot the magnitude responses $|S|$, $|T|$, $|KS|$, $|SP|$. (f) Use MATLAB to determine the upward gain margin $\uparrow GM$, downward gain margin $\downarrow GM$, phase margin PM, and delay margin DM for your design? (g) Plot the response (output y and control u) to a unit step reference command. Explain why your design achieves the steady state specification. Give the main reason for the overshoot in the output y. How is the response enhanced if you use a command pre-filter $W = \frac{z_k}{s+z_k}$ where $-z_k$ is a zero of the compensator K. (h) Discuss what happens in (d)-(g) as z is decreased toward $p = 1$ from above. What can be said about near right half plane pole-zero cancelations? ■

Problem 7 (SVD) Consider the matrix $M = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 10 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix}$. Plot $y = Mx$ in the $y_1 y_2$ plane when x lies on the unit circle $||x|| = 1$. Clearly discuss your plot using SVD concepts. ■