Chapter 1: Model Based Compensators and the Separation Principle

EE482 Exam 1
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Problem 1.3.11 (State Space Representations)
(a) Suppose \( H(s) = \frac{4s^2 + 8}{s^3 + 5s + 6} \). Determine a state space representation for \( H \).
(b) Determine a block diagram and state space representation for the following dynamical system:

\[
\begin{align*}
\dot{x}_1 &= x_1 + u + d \\
\dot{x}_2 &= e \\
u &= x_2 + 2e \\
e &= r - y \\
y &= x_1
\end{align*}
\] (1.82)

What are the closed loop poles?

Problem 1.3.12 (Transfer Function Matrix, Matrix Exponential, Modal Analysis)
Consider the dynamical system \( \dot{x} = Ax + Bu, y = Cx \), where

\[
A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -8 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \] (1.83)

(a) Determine the system transfer function from \( u \) to \( y \). What are the system poles?
(b) Use eigenvalue-eigenvector techniques to determine \( e^{At} \).
(c) Show how each mode can be excited by appropriate selection of the initial condition \( x_0 \).

Problem 1.3.13 (Laplace, Step Response)
Consider the dynamical system \( \dot{x} = Ax + Bu \), where

\[
A = \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \] (1.84)

Determine \( x \) when \( u \) is a unit step function.

Problem 1.3.14 (Transmission Zeros)
Consider the dynamical system \( \dot{x} = Ax + Bu, y = Cx + Du \) where

\[
A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 1 \\ 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \] (1.85)

(a) Determine the system transfer function matrix. What are the system poles?
(b) Determine the system’s transmission zeros. Hint: Only has one.
(c) Determine an associated input direction \( u_0 \).
(d) Determine an associated state direction \( x_0 \).

Problem 1.3.15 (Model Based Compensators)
Consider a SISO plant \( P = \frac{1}{s+2} \). Determine a model based compensator \( K \) such that the closed loop system
(1) has poles at \( s = -1 \pm j1, s = -100, -100 \), (2) follows step reference commands.