EEE 482 Final Exam - Diagnostics
Fall 2001

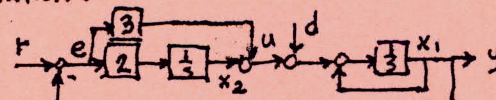
A. A. Rodriguez
GWC 612, 965-3712

Rules: Closed books, 1 $8\frac{1}{2}" \times 11" \times 0"$ sheet, No calculators, Open minds!

Problem #1 $\boxed{\text{State Space Representations}}$

a) Suppose $H(s) = \dfrac{s^2+1}{s(s+1)(s+2)}$. Determine a state space representation.

b) Determine a state space representation from (r, d) to y:



Problem #2 $\boxed{\text{Transfer Function Matrix, Matrix Exponential, Modal Analysis}}$

Consider the dynamical system $\dot{x} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & 3 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}_{B} u \quad y = \underbrace{[1 \ 0 \ 0]}_{C} x$

a) Determine the system transfer function from u to y.

b) Use eigenvalue-eigenvector techniques to determine e^{At}.

c) Show how each mode can be excited by appropriate selection of the initial condition x_0.

Problem #3 $\boxed{\text{Laplace, Step Response}}$

Consider the dynamical system $\dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$. Determine x when u is a unit step.

Problem #4 $\boxed{\text{Transmission Zeros}}$

Consider the dynamical system
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \hline y_1 \\ y_2 \end{bmatrix} = \left[\begin{array}{cc|cc} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ \hline -1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 0 \end{array}\right] \begin{bmatrix} x_1 \\ x_2 \\ \hline d_1 \\ d_2 \end{bmatrix}$$

a) Determine the system transfer function matrix H_{dy}.

b) Determine the system's transmission zeros.
Hint: Only has one zero.

c) Determine the associated input direction d_1, d_2.
Hint: Examine $H(z)$.

d) Determine state direction x_1, x_2.

Problem #5 $\boxed{\text{Model Based Compensator}}$

Consider a SISO plant $P = \frac{1}{s-1}$. Determine a fourth order compensator K such that closed loop system (1) has poles at $s = -1 \pm j1$, $s = -100$, $s = -100$ (2) follows step reference commands.

Problem #6 $\boxed{\text{Controllability \& Observability}}$

Consider the two LTI systems: $S_1 - \ \dot{x}_1 = -4x_1 + u_1$, $y_1 = -6x_1 + u_1$ $\quad S_2 - \ \dot{x}_2 = 2x_2 + u_2$, $y_2 = x_2$

Discuss the controllability, stabilizability, observability, and detectability properties of $S_1 S_2$ & $S_2 S_1$.

Fix

Problem #7 $\boxed{\text{State Transfer and State Construction}}$

Consider the dynamical system $\dot{x} = x + u \quad y = x + u$.

a) Determine a control law (minimum energy) which transfers the state from $x(0) = 1$ to $x(1) = 2$.

b) Given u & y on $t \in [0, 1]$, show how to determine $x(0) = x_0$.

+90°

Problem #8 $\boxed{\text{Steady State Analysis of a Transfer Function Matrix Using SVD}}$

Suppose that $H(j\omega_0) = U \Sigma V^H$ where $U = \begin{bmatrix} \frac{1}{\sqrt{2}} e^{j30^\circ} & \frac{1}{\sqrt{2}} e^{j60^\circ} \\ \frac{1}{\sqrt{2}} e^{j45^\circ} & -\frac{1}{\sqrt{2}} e^{j75^\circ} \end{bmatrix}$ $\Sigma = \begin{bmatrix} 10 & \\ & 1 \end{bmatrix}$ $V = \begin{bmatrix} \frac{\sqrt{19}}{10} e^{-j45^\circ} & \frac{9}{10} e^{j25^\circ} \\ \frac{9}{10} e^{j20^\circ} & -\frac{\sqrt{19}}{10} e^{-j40^\circ} \end{bmatrix}$

What sinusoidal input maximizes $z = \dfrac{\|h * u\|_2}{\|u\|_2}$?

75−45

What is corresponding output? $\quad$ 60−30° $\quad$ +45+25 $\quad$ 90−20

What sinusoidal input minimizes z?
What is corresponding output?

Problem #9 $\boxed{\text{LQG/LTRO}}$ Assume we have a plant $P = [A_p, B_p, C_p]$ with A_p invertible. Suppose we want to design a stabilizing compensator K which (1) permits step commands to be followed, (2) which guarantees closed loop stability, & (3) which has nice robustness properties at the plant output. Show how such a controller may be designed using the LQG/LTRO method. Discuss how you would match singular values at low & at high frequencies. Provide all relevant equations & explanations. Give sufficient conditions for LTR. Summarize properties that can be achieved when LTR is possible. Demonstrate ideas for $A_p = B_p = C_p = 1$. Sketch a block diagram for the MBC which exhibits its state feedback & state estimation substructures.