EEE582 Exam 2, Fall 2012 A.A. Rodriguez
Rules: One 8.5 $\times$ 11 sheet permitted, calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Gaussian Elimination, Fundamental Subspaces, General Solution, Projections) Consider the system of linear algebraic equations $Ax = b$ where $A = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 2 & 2 & 2 & 2 & 2 \\ 1 & 1 & 1 & 1 & 2 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.
(a) Parameterize the set of all solutions x.
(b) Determine a basis for each of the four (4) fundamental subspaces: $\mathcal{R}(A)$, $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$.
(c) Given that $b = [\ b_1 \ \ b_2 \ \ b_3\ ]^T$ does not lie within the range space of A, determine the set of all x which minimizes the Euclidean norm (distance) $\|b - Ax\|$. Determine the projection of b onto $\mathcal{R}(A)$.
(d) For the solutions in (c), determine one that lies within $\mathcal{R}(A^T)$. What can be said about such a solution?

Problem 2 (State Space Realization and Arithmetic) (a) Provide two distinct state space realizations for the following LTI system

$$H(s) = \frac{c_3 s^3 + c_2 s^2 + c_1 s + c_o}{s^3 + a_2 s^2 + a_1 s + a_o}. \tag{1}$$

Sketch a block diagram for each realization. Indicate all state variables on your diagrams.
(b) Determine a state space representation for the feedback system with external signals (r, d_i, d_o, n), state $x = [\ x_p \ \ x_k\ ]^T$, and outputs (y, u, e) defined by the following equations:

$$e = r - y - n \qquad u_p = u + d_i \tag{2}$$
$$\dot{x}_k = A_k x_k + B_k e \qquad \dot{x}_p = A_p x_p + B_p u_p \tag{3}$$
$$u = C_k x_k \qquad y = C_p x_p + d_o. \tag{4}$$

Problem 3 (Modal Analysis) Consider the linear dynamical system $\dot{x} = Ax$, $x(0) = x_o$ with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$. (a) Determine the system's characteristic equation? (b) Determine all eigenvalues of A. (c) Determine a set of linearly independent eigenvectors for A. Is A diagonalizable? (d) Compute x. (e) Determine how each mode can be independently excited via carefully selected (real) initial conditions. Compute the response for each initial condition.

Problem 4 (State Computation) Consider the linear system $\dot{x} = Ax + Bu$, $y = Cx$ with $A = \begin{bmatrix} -10 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} -9 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$. (a) Sketch a block diagram for the system. (b) Compute x and y when $u = \delta(t)$ (unit impulse function) and $x(0) = [\ 0 \ \ 0 \ \ 0\ ]^T$.

Problem 5 (Transmission Zeros) Consider the linear time invariant (LTI) system $\dot{x} = Ax + Bu$, $y = Cx + Du$ with $A = \begin{bmatrix} -2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -2 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 1 & -1 & 0 \\ 1 & 0 & -2 & 1 \end{bmatrix}$. (a)Sketch a block diagram for the system. (b) Determine the system transfer function matrix $H(s)$. (c) Determine the associated system poles. (d) Determine the associated system transmission zeros and the associated input and state directions. (e) Use the system block diagram to illustrate the significance of the associated input and state directions. Show how the signal information propagates through the system.

Problem 6 (Controllability, Reachability, State Transfer, State Feedback) Consider the LTI system defined by the state space representation $\dot{x} = Ax + Bu$ with $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & -2 & 0 \\ 1 & -2 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.
(a) Sketch a block diagram for the system.

(b) Determine all system eigenvalues and an associated set of right and left eigenvectors.
(c) Is the system controllable? Explain your answer using multiple tests.
(d) Determine the set of all states that are reachable from $x(0) = 0$?
(e) What is the set of all closed loop poles that are achievable via full state feedback? Explain.
(f) What is the range of the controllability matrix? Relate your answer to your answer in (d).
(g) Can $x(1) = [\ 1 \quad 1 \quad 1\ ]^T$ be reached from $x(0) = 0$? If so, determine the minimum energy control law which performs the transfer? In such a case, is this the only control law that will perform the transfer? If not, determine what closest state $x(1)$ can be reached and determine the minimum energy control law which performs the transfer. In such a case, is your control law the only control law that can perform the transfer?
(h) Can $x(1) = [\ 0 \quad 0 \quad 1\ ]^T$ be reached from $x(0) = 0$? If so, determine the minimum energy control law which performs the transfer? In such a case, is this the only control law that will perform the transfer? If not, determine what closest state $x(1)$ can be reached and determine the minimum energy control law which performs the transfer. In such a case, is your control law the only control law that can perform the transfer?

Problem 7 (Observability, Reconstruction, Observers)
Consider the LTI system from Problem 6 with $y = x_3$ initially. (a) Is the system observable? Explain your answer using multiple tests.
(b) Given y and u on $[0, t_f]$, can you determine the initial state x_o uniquely? If so, discuss how. If not, explain why not. In such a case, show how knowledge of y and its derivatives at $t = 0$ can be used to determine the set of all possible x_o. In such a case, how does your result relate to the eigenvectors of A.
(c) Consider a model based observer. What is the set of all achievable closed loop poles? Explain.

Now suppose that $y = x_1$.

(d) Is the system observable? Explain your answer using multiple tests.
(e) Given y and u on $[0, t_f]$, can you determine the initial state x_o uniquely? If so, discuss how. If not, explain why not. In such a case, show how knowledge of y and its derivatives at $t = 0$ can be used to determine the set of all possible x_o. In such a case, how does your result relate to the eigenvectors of A.
(f) Consider a model based observer. What is the set of all achievable closed loop poles? Explain.
(g) Can the observability properties of the system be changed via full state feedback? Explain.

Problem 8 (Model Based Compensator)
Consider the linear time invariant (LTI) plant $P(s) = \frac{10-s}{s}$ with $A_p = 0$, $B_p = 1$, $C_p = 10$, $D_p = -1$ and variables (u_p, x_p, y_p). (a) Design a model based compensator which satisfies the following design specifications: (i) zero steady state error to step input disturbances d_i, (ii) closed loop poles at $s = -1 \pm j1$, $s = -20 \pm j20$. What is the associated (approximate) closed loop settling time? (b) Discuss how you would reduce overshoot to step reference commands.