EEE582 Exam 1, Fall 2013 A.A. Rodriguez
Rules: One 8.5 × 11 sheet permitted, calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Gaussian Elimination, Fundamental Subspaces, General Solution, Projections)
Consider the system of linear algebraic equations:

$$Ax = b$$

with $A = \begin{bmatrix} 1 & 2 & 1 & 2 & 3 \\ 2 & 3 & 2 & 4 & 6 \\ 3 & 5 & 3 & 6 & 9 \\ 4 & 6 & 4 & 8 & 12 \end{bmatrix}$ and $b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{bmatrix}$.

(a) Parameterize the set of all solutions x.

(b) Determine a basis for each of the four (4) fundamental subspaces: $\mathcal{R}(A)$, $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$.

(c) Determine a solution that lies within the range space of A^T. What can be said about such a solution?

(d) Given that $b = [\, b_1 \;\; b_2 \;\; b_3 \;\; b_4 \,]^T$ does not lie within the range space of A, determine the set of all x which minimizes the Euclidean norm (distance) $\|b - Ax\|$. Amongst these, how would you determine the minimum norm solution?

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Problem 2 (State Space Realization and Arithmetic)
(a) Provide two distinct state space realizations for the following LTI system

$$H(s) = \frac{15s^3 + 45}{5s^3 - 60}. \tag{1}$$

Sketch a block diagram for each realization. Indicate all state variables on your diagrams.
(b) Determine a state space representation for the feedback system with external signals (r, d_i, d_o, n), state $x = [\, x_p \;\; x_k \,]^T$, and outputs (e, u, y) defined by the following equations:

$$e = r - y - n \qquad u_p = u + d_i \tag{2}$$
$$\dot{x}_k = A_k x_k + B_k e \qquad \dot{x}_p = A_p x_p + B_p u_p \tag{3}$$
$$u = C_k x_k + D_k e \qquad y = C_p x_p + D_p u_p + d_o. \tag{4}$$

Assume that $M \stackrel{\text{def}}{=} (I + D_p D_k)^{-1}$ is well defined. ■

Problem 3 (Modal Analysis)
Consider the linear dynamical system:

$$\dot{x} = Ax \qquad x(0) = x_o$$

with $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 10 \\ 1 & -10 & 0 \end{bmatrix}$. (a) Determine the system's characteristic equation? (b) Determine all eigenvalues of A. (c) Determine a set of linearly independent eigenvectors for A. Is A diagonalizable? (d) Compute x. (e) Determine how each mode can be independently excited via carefully selected (real) initial conditions. Determine the response for each initial condition. ■

Problem 4 (State Computation)
Consider the linear system:

$$\dot{x} = Ax + Bu \qquad y = Cx$$

with $A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 & 10 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$. (a) Sketch a block diagram for the system. (b) Compute x and y when $u = \delta(t)$ (unit impulse) and $x(0) = 0$. ■

Problem 5 (Transmission Zeros)
Consider the linear time invariant (LTI) system:

$$\dot{x} = Ax + Bu \qquad\qquad y = Cx + Du$$

with $A = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$. (a)Sketch a block diagram for the system. (b) Determine the system transfer function. (c) Determine the associated system poles. (d) Determine the associated system transmission zeros and the associated input and state directions. (e) Use the system block diagram to illustrate the significance of the associated input and state directions. Show how information is propagated through the system. ■

Problem 6 (Controllability)
Consider the LTI system defined by the state space representation:

$$\dot{x} = Ax + Bu \qquad\qquad y = Cx$$

with $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 0 & 1 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1-a \\ 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 1 \end{bmatrix}$ where the parameter a is specified below.
(a) Sketch a block diagram for the system.
Suppose $a = 0$.
(b) Is the system controllable? Explain your answer using multiple tests.
(c) Determine the set of all states that are reachable from $x(0) = 0$?
Is $x(1) = [\,0 \;\; 1 \;\; 0\,]^T$ reachable from $x(0) = 0$? Explain. If not, determine the closest reachable state.
Is $x(1) = [\,2 \;\; -2 \;\; 1\,]^T$ reachable from $x(0) = 0$? Explain. If not, determine the closest reachable state.
(d) Show how to construct a minimum energy control law which transfers the state from $x(0) = 0$ to a reachable state $x(t_f) = x_f$. Can another control law achieve the transfer? If yes, show how to construct one.
(e) What closed loop poles can be achieved with full state feedback? Explain.

Now suppose $a = 1$.
(f) Is the system controllable? Explain your answer using multiple tests.
(g) Determine the set of all states that are reachable from $x(0) = 0$?
Is $x(1) = [\,0 \;\; 1 \;\; 0\,]^T$ reachable from $x(0) = 0$? Explain. If not, determine the closest reachable state.
Is $x(1) = [\,2 \;\; -2 \;\; 1\,]^T$ reachable from $x(0) = 0$? Explain. If not, determine the closest reachable state.
(h) Show how to construct a minimum energy control law which transfers the state from $x(0) = 0$ to a reachable state $x(t_f) = x_f$. Can another control law achieve the transfer? If yes, show how to construct one.
(i) What closed loop poles can be achieved with full state feedback? Explain.
HOME:
(j) Explain why using an open loop control u as above may be highly undesirable in practice.
Hint: When is a feedback control law essential?
Suppose $a = 0$ or $a = 1$.
(k) Is the system observable? Explain your answer using multiple tests.
(l) Determine a modal expression for y when $u = 0$ and $x(0) = x_o$.
(m) In (l), can $x(0) = x_o$ be determined uniquely from y? Explain.

(n) Relate y and its first $n-1$ derivatives at $t=0$ to x_o. Determine the general solution x_o. Is it unique? Determine a solution x_o that lies in the row space of the Observability matrix. What can be said about this x_o?
(o) Is the above derivative method a good method for determining x_o? Explain. What would be a better method. Explain. Hint: Show how to construct a state estimator/observer.
(p) For the system above, discuss how a matrix H can be used to place the eigenvalues of $A-HC$. Clearly point out any limitations that may exist. ■

Problem 7 (Model Based Compensator)
Consider the linear time invariant (LTI) plant

$$P(s) = \frac{z-s}{s-p}$$

with $z, p > 0$, $A_p = p$, $B_p = z-p$, $C_p = 1$, $D_p = -1$ and variables $(u_p,\ x_p,\ y_p)$. (a) Design a model based compensator which satisfies the following design specifications:
(i) zero steady state error to step reference commands,
(ii) closed loop poles at $s = -1 \pm j1$, $s = -20 \pm j20$. ■

HOME:
(b) Determine the final compensator K.
Let $p = 1$ and consider $z = 10, 5, 2$. For each of these systems, do the following.
(c) Use MATLAB to verify that your design achieves the desired closed loop poles. (d) Let $L \stackrel{\text{def}}{=} PK$ denote the open loop transfer function. Use MATLAB to sketch Bode magnitude and phase plots for L. (e) Let $S = \frac{1}{1+L}$, $T = 1 - S = \frac{L}{1+L}$. Use MATLAB to plot the magnitude responses $|S|$, $|T|$, $|KS|$, $|SP|$. (f) Use MATLAB to determine the upward gain margin $\uparrow GM$, downward gain margin $\downarrow GM$, phase margin PM, and delay margin DM for your design? (g) Plot the response (output y and control u) to a unit step reference command. Explain why your design achieves the steady state specification. Give the main reason for the overshoot in the output y. How is the response enhanced if you use a command pre-filter $W = \frac{z_k}{s+z_k}$ where $-z_k$ is a zero of the compensator K. (h) Discuss what happens in (d)-(g) as z is decreased toward $p = 1$ from above. What can be said about near right half plane pole-zero cancelations?