EEE582 Final Exam, Fall 2006 A.A. Rodriguez
Rules: Closed notes/books, No calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (State Space Representation)
Consider a unity feedback system defined by the equations

$$\dot{x}_p = A_p x_p + B_p u_p \qquad u_p = d_i + u \qquad u = C_k x_k \tag{1}$$
$$\dot{x}_k = A_k x_k + B_k e \qquad e = C_w x_w - n - y \qquad \dot{x}_w = A_w x_w + B_w r \tag{2}$$
$$y = d_o + y_p \qquad y_p = C_p x_p \tag{3}$$

Sketch a block diagram for the above feedback system. Determine a state space representation for the system with inputs (r, d_i, d_o, n), states $(x_p,\ x_k,\ x_w)$, and outputs $(y,\ u,\ e)$. ∎

Problem 2 (Gaussian Elimination, Fundamental Spaces, Least Squares, Minimum Norm)
Consider the following linear algebraic system of equations:

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \tag{4}$$

(a) Determine the general solution.
(b) Determine a basis for the four fundamental subspaces: $R(A), R(A^H), N(A), N(A^H)$.

Now let $b = [\ 1 \ \ -1 \ \ 1\ ]^T$ and consider the vector norm $\|z\| \stackrel{\text{def}}{=} \sqrt{z^T z}$.
(c) Determine the set of all possible x that minimizes the error $\|b - Ax\|_2$.
(d) Determine the projection $P_{R(A)}b$ of the vector b onto the range of A.
(e) Determine the minimum norm x which minimizes the error $\|b - Ax\|_2$. ∎

Problem 3 (SVD)
Construct a matrix M which
(1) maps the unit vector $v_1 = \left[\ \frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\ \right]^T$ to the vector $10u_1 = 10\left[\ \frac{1}{\sqrt{2}} \ \ -\frac{1}{\sqrt{2}}\ \right]^T$,
(2) maps the unit vector $v_2 = \left[\ -\frac{1}{\sqrt{2}} \ \ \frac{1}{\sqrt{2}}\ \right]^T$ to the vector $0.1u_2 = 0.1\left[\ -\frac{1}{\sqrt{2}} \ \ -\frac{1}{\sqrt{2}}\ \right]^T$.
Show pictorially how the matrix M maps the unit circle onto an ellipse. ∎

Problem 3 (Transmission Zeros)
Consider the LTI system with state space representation $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & \alpha \\ 0 & 1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ where $\alpha \neq -1$. (a) Sketch a block diagram for the system. (b) Determine the transfer function matrix $H(s)$. (c) Determine all finite transmission zeros and associated input and state directions. (d) Show how the energy propagates through the system. ∎

Problem 4 (Controllability, State Transfer, Pole Placement)
Consider the LTI system defined by the state space pair $A = \begin{bmatrix} -1 & 2 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
(a) Sketch a block diagram for the system. Identify two key subsystems within the system. Determine their transfer functions. Are there any pole-zero cancellations?
(b) Is the system controllable? Explain your answer. Are there any pole-zero cancellations?
(c) Does there exist a control law which will transfer the state of the system from $x(0) = x_o = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$ to $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$? If so, discuss how to determine one. Also, determine a minimum energy state transferring control law.
(d) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = -1, -10$? If so, determine a suitable control gain matrix G. If not, explain why. ∎

Problem 5 (Controllability, State Transfer, Pole Placement)
Consider the LTI system defined by the state space pair $A = \begin{bmatrix} -1 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
(a) Sketch a block diagram for the system. Identify two key subsystems within the system. Determine their transfer functions. Are there any pole-zero cancellations?
(b) Is the system controllable? Explain your answer. Are there any pole-zero cancellations?
(c) Assuming $x_o = 0$, determine an expression for $x(t)$ using modal analysis concepts. Determine a basis for the set of states that are reachable from $x_o = 0$.
(d) Does there exist a control law which will transfer the state of the system from $x(0) = x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x(1) = \begin{bmatrix} 10 \\ 10 \end{bmatrix}$? If so, determine one. Also, determine a minimum energy state transferring control law.
(e) Does there exist a control law which will transfer the state of the system from $x(0) = x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ to $x(1) = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$? If so, determine one. If not, determine what state closest to $x(1) = \begin{bmatrix} 4 \\ 10 \end{bmatrix}$ is reachable. Then determine a control law which will achieve the transfer to this reachable state. Is your control law unique? If not, show how to determine another state transferring control law and determine the minimum energy state transferring control law.
(f) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = -2, -3$? If so, determine a suitable control gain matrix G. If not, explain why. ■

Problem 6 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -3 & 1 \\ 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [\,2 \;\; 1\,]$.
(a) Sketch a block diagram for the system. Identify two key subsystems within the system. Determine their transfer functions. Are there any pole-zero cancellations?
(b) Is the system observable? Explain your answer. Are there any pole-zero cancellations?
(c) Suppose that $u = 0$ and y is known on $t \in [0, 1]$. Determine an expression for the set of all possible initial conditions x_o. Can x_o be determined uniquely? Explain. Determine the minimum norm initial condition.
(d) Can one design an observer with closed loop poles at $s = -1, -2$? If not, explain why. If so, determine a suitable observer gain matrix H. Moreover, determine the state estimation error $\tilde{x} \stackrel{\text{def}}{=} x - \hat{x}$ when $u = 0$, the initial system state is $x_o = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and the initial state estimate (used in your observer) is $\hat{x}_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. ■

Problem 7 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -1 & 1 \\ 0 & -4 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [\,3 \;\; 1\,]$.
(a) Sketch a block diagram for the system. Identify two key subsystems within the system. Determine their transfer functions. Are there any pole-zero cancellations?
(b) Is the system observable? Explain your answer. Are there any pole-zero cancellations?
(c) Suppose that $u = 0$ and y is known on $t \in [0, 1]$. Determine an expression for y. Determine the set of all possible initial conditions x_o. Can x_o be determined uniquely? Explain. Determine the minimum norm initial condition.
(d) Can one design an observer with closed loop poles at $s = -1, -5$? If so, determine a suitable observer gain matrix H. If not, explain why.
(e) Can one design an observer with closed loop poles at $s = -2, -5$? If so, determine a suitable observer gain matrix H. If not, explain why. ■

Problem 8 (Model Based Compensator Design)
Consider the linear time invariant plant

$$P(s) = 10$$

with state space quadruple $A_p = 0$, $B_p = 0$, $C_p = 0$, $D_p = 10$.
(a) Show how to design a model based compensator which satisfies the following design specifications:
(i) zero steady state error to ramp reference commands,
(ii) closed loop poles at $s = -1 \pm j1$, $s = -100 \pm j100$.
(b) Discuss how one might minimize the overshoot due to a step reference command. ■