EEE582 Final, Spring 2005 A.A. Rodriguez

Rules: Closed notes/books, 1 sheet permitted, No calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Closed Loop System: State Space Representation)

Consider a negative feedback system defined by the equations: $y = P(u + d)$, $u = Ke$, $e = Wr - y$ *where* $P = [A_p, B_p, C_p]$ *has state* x_p , $K = [A_k, B_k, C_k]$ *has state* x_k, *and* $W = [A_w, B_w, C_w]$ *has state* x_w. *Sketch a block diagram of the feedback system. Determine a state space representation for the closed loop system with exogenous signals* (r, d), *states* $x = [\ x_p^T\ x_k^T\ x_w^T\ ]^T$, *and outputs* (e, y).

Problem 2 (Modal Analysis)

Consider the linear dynamical system:

$$\dot{x} = Ax \qquad x(0) = x_o$$

with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix}$. *Determine eigenvalues and eigenvectors for the system. Is A diagonalizable? Determine how each mode can be excited independently via carefully selected initial conditions. Determine the response for each initial condition.*

Problem 3 (Transmission Zeros)

Consider the linear system:

$$\dot{x} = Ax + Bu \qquad y = Cx$$

with $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [\ 1 \quad 0\ ]$, $D = 1$. *(a) Determine the system transfer function. (b) Determine the associated transmission zeros and the associated input and state directions. (c) Sketch a block diagram for the system and use it to illustrate the significance of the associated input and state directions. Note: Only real inputs and initial conditions are permitted.*

Problem 4 (Controllability, State Transfer, Pole Placement)

Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = [\ 1 \quad 0\ ]$.

(a) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$? *If so, show how to determine the minimum energy state transferring control law; i.e. specify the appropriate formula. Will other control laws accomplish the state transfer? Explain clearly!*

(b) Does there exist a full state feedback control law $u = -Gx$ *such that the closed loop system has poles at* $s = -1 \pm j1$? *If so, show how to determine a suitable control gain matrix* G. *If not, explain why.*

Problem 5 (Controllability, State Transfer, Pole Placement)

Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -3 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1/3 \\ 1 \end{bmatrix}$, $C = [\ 1 \quad 0\ ]$.

(a) Is the system controllable? Explain your answer.

(b) Assuming $x_o = 0$, *determine an expression for* $x(t)$ *using modal analysis concepts. Determine a basis for the set of states that are reachable from* $x_o = 0$.

(c) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$? *If so, show how to determine the minimum energy state transferring control law.*

(d) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$? *If so, determine one. If not, determine what state closest to* $x(1) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}$ *is reachable. Then determine*

a minimum energy control law which will achieve the transfer to this reachable state. Is your control law unique? Explain.
(e) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = -1, -2$? If so, show how to determine a suitable control gain matrix G. If not, explain why.
(f) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = -3, -4$? If so, show how to determine a suitable control gain matrix G. If not, explain why. ∎

Problem 6 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined in Problem 4.
(a) Is the system observable? Explain your answer.
(b) Suppose that $u = 0$ and y is known on $t \in [0, 1]$. Determine an expression for the set of all possible initial conditions x_o. Can x_o be determine uniquely? Explain. Determine the minimum norm initial condition.
(c) Can you design an observer with closed loop poles at $s = -1, -2$? If not, explain why. If so, show how to determine a suitable observer gain matrix H. ∎

Problem 7 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -3 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 1/3 & 1 \end{bmatrix}$.
(a) Is the system observable? Explain your answer.
(b) Suppose that $u = 0$ and $y = 4$ on $t \in [0, 1]$. Determine an expression for the set of all possible initial conditions x_o. (HINT: Use modal ideas.) Can x_o be determined uniquely? Explain. Determine the minimum norm initial condition.
(c) Can you design an observer with closed loop poles at $s = -1, -2$? If so, show how to determine a suitable observer gain matrix H. If not, explain why.
(e) Does there exist an observer with closed loop poles at $s = -2, -3$? If so, show how to determine a suitable observer gain matrix H. If not, explain why. ∎

Problem 8 (Model Based Compensator Design)
Consider the unstable plant $P = \frac{2-s}{s-1} = [A_p, B_p, C_p, D_p]$ where $A_p = 1$, $B_p = 1$, $C_p = 1$, $D_p = -1$. Design a model based compensator K which guarantees zero steady state error to step reference commands and places closed loop poles at $s = -1 \pm j1, -10 \pm j10$. ∎

Problem 9 (Application of SVD)
Suppose that the SVD for a stable transfer function matrix is given by $H(j6) = U\Sigma V^H$ where

$$U = \begin{bmatrix} 0.9275\, e^{j44.1583^\circ} & 0.3739\, e^{j57.6869^\circ} \\ 0.3739\, e^{-j110.8821^\circ} & 0.9275\, e^{j82.6465^\circ} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 1.5956 & 0.0000 \\ 0.0000 & 0.7736 \end{bmatrix} \tag{1}$$

$$V = \begin{bmatrix} 0.9433\, e^{j0.0000^\circ} & 0.3320\, e^{j180.0000^\circ} \\ 0.3320\, e^{j85.9345^\circ} & 0.9433\, e^{j85.9345^\circ} \end{bmatrix}. \tag{2}$$

Determine the minimally amplified sinusoidal input vector u and the associated steady state output y. ∎