EEE582 Exam 2, Spring 2005 A.A. Rodriguez

Rules: Closed notes/books, 1 sheet permitted, No calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Closed Loop System: State Space Representation)

Consider a negative feedback system defined by the equations: $y = P(u+d)$, $u = Ke$, $e = r - y$ *where* $P = [A_p, B_p, C_p]$ *and* $K = [A_k, B_k, C_k]$. *Sketch a block diagram of the feedback system. Determine a state space representation for the closed loop system with exogenous signals* (r, d) *and outputs* (e, y). ■

Problem 2 (Application of SVD)

Suppose that the SVD for a stable transfer function matrix is given by $H(j10) = U\Sigma V^H$ *where*

$$U \;+= \begin{bmatrix} 0.9275\, e^{j44.1583^\circ} & 0.3739\, e^{j57.6869^\circ} \\ 0.3739\, e^{-j110.8821^\circ} & 0.9275\, e^{j82.6465^\circ} \end{bmatrix} \qquad \Sigma = \begin{bmatrix} 1.5956 & 0.0000 \\ 0.0000 & 0.7736 \end{bmatrix} \tag{1}$$

$$V = \begin{bmatrix} 0.9433\, e^{j0.0000^\circ} & 0.3320\, e^{j180.0000^\circ} \\ 0.3320\, e^{j85.9345^\circ} & 0.9433\, e^{j85.9345^\circ} \end{bmatrix}. \tag{2}$$

Determine the maximally amplified sinusoidal input vector u *and the associated steady state output* y. ■

Problem 3 (Controllability, State Transfer, Pole Placement)

Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

(a) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$*? If so, show how to determine the minimum energy state transferring control law. Will other control laws accomplish the state transfer? Explain.*

(b) Does there exist a full state feedback control law $u = -Gx$ *such that the closed loop system has poles at* $s = -1 \pm j1$*? If so, determine a suitable control gain matrix* G*. If not, explain why.* ■

Problem 4 (Controllability, State Transfer, Pole Placement)

Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$.

(a) Is the system controllable? Explain your answer.

(b) Assuming $x_o = 0$, *determine an expression for* $x(t)$ *using modal analysis concepts. Determine a basis for the set of states that are reachable from* $x_o = 0$.

(c) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$*? If so, show how to determine the minimum energy state transferring control law.*

(d) Does there exist a control law which will transfer the state of the system from $x_o = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ *to* $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$*? If so, determine one. If not, determine what state closest to* $x(1) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ *is reachable. Then determine a control law which will achieve the transfer to this reachable state. Is your control law unique? Explain.*

(e) Does there exist a full state feedback control law $u = -Gx$ *such that the closed loop system has poles at* $s = -2, -3$*? If so, determine a suitable control gain matrix* G*. If not, explain why.*

(f) Does there exist a full state feedback control law $u = -Gx$ *such that the closed loop system has poles at* $s = -3, -4$*? If so, determine a suitable control gain matrix* G*. If not, explain why.* ■

Problem 5 (Observability, State Reconstruction, Pole Placement)

Consider the LTI system defined in Problem 3.

(a) Is the system observable? Explain your answer.

(b) Suppose that $u = 0$ *and* y *is known on* $t \in [0, 1]$. *Determine an expression for the set of all possible initial conditions* x_o. *Can* x_o *be determine uniquely? Explain. Determine the minimum norm initial condition.*

(c) Can you design an observer with closed loop poles at $s = -1, -3$*? If not, explain why. If so, determine a suitable observer gain matrix* H. ■

Problem 6 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system defined by the state space triple $A = \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0.5 & 1 \end{bmatrix}$.
(a) Is the system observable? Explain your answer.
(b) Suppose that $u = 0$ *and* y *is known on* $t \in [0, 1]$. *Determine an expression for the set of all possible initial conditions* x_o. *Can* x_o *be determine uniquely? Explain. Determine the minimum norm initial condition.*
(c) Can you design an observer with closed loop poles at $s = -2, -3$? *If so, show how to determine a suitable observer gain matrix* H. *If not, explain why.*
(e) Does there exist an observer with closed loop poles at $s = -3, -4$? *If so, show how to determine a suitable observer gain matrix* H. *If not, explain why.* ■

Problem 7 (Model Based Compensator Design)
Consider the unstable plant $P = \frac{10-s}{s-1} = [A_p, B_p, C_p, D_p]$ *where* $A_p = 1$, $B_p = 1$, $C_p = 9$, $D_p = -1$. *Design a model based compensator* K *which guarantees zero steady state error to step reference commands and places closed loop poles at* $s = -1 \pm j1, -10 \pm j10$. ■