EEE582 Exam 1, Spring 2005
Rules: Closed notes/books, calculators permitted, open minds.

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Problem 1 (Gaussian Elimination, Fundamental Subspaces, General Solution)
Consider the linear system:

$$Ax = b$$

with $A = \begin{bmatrix} 1 & 3 & 1 \\ 2 & 6 & 2 \end{bmatrix}$ *and* $b = \begin{bmatrix} 5 \\ 10 \end{bmatrix}$.

(a) Parameterize the set of all solutions $x \in \mathcal{R}^2$.
(b) Determine a solution that lies within the range space of A^T.
(c) Determine a basis for each of the four (4) fundamental subspaces: $\mathcal{R}(A)$, $\mathcal{R}(A^T)$, $\mathcal{N}(A)$, $\mathcal{N}(A^T)$. ■

Problem 2 (State Space Realization and Arithmetic)
(a) Provide two distinct state space realizations for the following LTI system

$$H(s) = \frac{20s^2 + 1}{10s^2 - 5}. \tag{1}$$

Sketch a block diagram for each realization. Indicate all state variables on your diagrams.
(b) Determine a state space representation for the following feedback system with external signals (r, d)*, state* $x = [\, x_p \quad x_k \,]^T$*, and output* e*:*

$$e = r - y \qquad u_p = u + d \tag{2}$$
$$\dot{x}_k = e \qquad \dot{x}_p = x_p + u_p \tag{3}$$
$$u = 5x_k + 5e \qquad y = x_p. \tag{4}$$

Determine the closed loop poles. ■

Problem 3 (Diagonizability, Matrix Exponential)
Consider the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$. *Is* A *diagonalizable? Compute* e^{At}. ■

Problem 4 (Modal Analysis)
Consider the linear dynamical system:

$$\dot{x} = Ax \qquad x(0) = x_o$$

with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -2 \end{bmatrix}$. *Determine eigenvalues and eigenvectors for the system. Is* A *diagonalizable? Determine how each mode can be excited independently via carefully selected initial conditions. Determine the response for each initial condition.* ■

Problem 5 (State Computation)
Consider the linear system:

$$\dot{x} = Ax + Bu$$

with $A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. *Compute* x *when* $u = 1(t)$ *and* $x(0) = [\, 1 \;\; 1 \,]^T$. ■

Problem 6 (Transmission Zeros)
Consider the linear system:

$$\dot{x} = Ax + Bu \qquad y = Cx$$

with $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 1 & 0 \end{bmatrix}$. *(a) Determine the system transfer function. (b) Determine the associated transmission zeros and the associated input and state directions. (c) Sketch a block diagram for the system and use it to illustrate the significance of the associated input and state directions.* ■

Problem 7 (Singular Value Decomposition)
Consider the linear system $y = Mw$ *where* $M = \begin{bmatrix} 2 & 3 \\ 3 & 2 \end{bmatrix}$. *(a) Determine an SVD for* M. *(b) Provide a carefully labeled sketch illustrating how the unit circle in the* w*-plane gets mapped into the* y*-plane.* ■

Problem 8 (Linearization)
Consider the following nonlinear model for a pendulum:

$$\dot{x} = f(x, u)$$

with $f(x, u) = \begin{bmatrix} x_2 \\ \sin(x_1) + u \end{bmatrix}$. *(a) Determine all system equilibria. (b) Linearize the above nonlinear system about the equilibrium associated with* $u_e = 0$ *and the pendulum vertical. (c) Discuss the stability of the resulting linear system. Discuss how you would excite each mode?* ■