EEE582 Final Exam, Fall 2009 A.A. Rodriguez
Rules: Closed notes/books, No calculators permitted, open minds. GWC 352, 965-3712

Problem 1 (Gaussian Elimination, Fundamental Spaces, Least Squares, Minimum Norm)
Consider the linear algebraic system of equations $Ax = b$ where:

$$A = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix} \qquad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \tag{1}$$

(a) Determine the general solution.
(b) Determine a basis for the four fundamental subspaces: $R(A), R(A^T), N(A), N(A^T)$.
(c) Assuming $b \in R(A)$, determine the minimum norm solution x.
(d) Determine the set of all possible x that minimizes the error $\|b - Ax\|_2$.
(e) Determine the projection $P_{R(A)}b$ of the vector b onto the range of A.
(f) Determine the minimum norm x which minimizes the error $\|b - Ax\|_2$. ∎

Problem 2 (LTI System Computation)

Consider the LTI system defined by: $A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$, $D = 0$.

(a) Sketch a block diagram for the system.
(b) Compute the system transfer function from u to y.
(c) Determine x when $u = \delta(t)$ and $x(0) = 0$. ∎

Problem 3 (Controllability, State Transfer, Pole Placement)

Consider the LTI system defined by: $A = \begin{bmatrix} 3 & 1 & 0 \\ 0 & 2 & -z \\ 0 & 0 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$, $D = 0$ where $z = 2$ initially.

(a) Sketch a block diagram for the system.
(b) Is the system controllable? Explain your answer in terms of controllability matrix, PBH, and pole-zero cancelations.
(c) Assume that $x(0) = 0$. Use modal concepts to determine an expression for $x(1)$.
(d) Does there exist a control law which will transfer the state of the system from $x(0) = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$ to $x(1) = \begin{bmatrix} 0 & 3 & 3 \end{bmatrix}$? If so, show how to determine the minimum energy state transferring control law. If not, determine the closest state that is reachable and determine a minimum energy control to reach that state.
(e) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = 2, -5, -6$? If so, show how to determine a suitable control gain matrix G. If not, explain why.
(f) Does there exist a full state feedback control law $u = -Gx$ such that the closed loop system has poles at $s = -4, -5, -6$? If so, show how to determine a suitable control gain matrix G. If not, explain why.
(g) How do your answers above change when $z = 5$? Just explain critical differences. ∎

Problem 4 (Observability, State Reconstruction, Pole Placement)
Consider the LTI system with state space (A, B, C, D) as given in Problem 3. Let $z = 0$ initially.
(a) Sketch a block diagram for the system.
(b) Is the system observable? Explain your answer. Are there any pole-zero cancelations?
(c) Suppose that $u = 0$ and y is known on $t \in [0, 1]$. Determine an expression for y using modal concepts.
(d) Suppose $u = 0$ and $y = ae^{2t} + be^{3t}$. Determine the set of all possible initial conditions x_o. Can x_o be determined uniquely? Explain.
(e) Determine the minimum norm initial condition x_o for the scenario given in (d).
(f) Can one design an observer with closed loop poles at $s = -4, -5, -6$? If not, explain why. If so, show how to determine a suitable observer gain matrix H.
(g) Can one design an observer with closed loop poles at $s = 0, -5, -6$? If not, explain why. If so, show how to determine a suitable observer gain matrix H.
(h) Show how you would compute the state estimation error $\tilde{x} \stackrel{\text{def}}{=} x - \hat{x}$ when $u = 0$.
(i) How do your answers above change when $z = 5$? Just explain critical differences. ∎

Problem 5 (Transmission Zeros)

Consider the LTI system defined by: $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $C = \begin{bmatrix} -1 & 2 \\ -2 & -1 \end{bmatrix}$, $D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(a) Sketch a block diagram for the system.
(b) Compute the system transfer function matrix.
(c) Determine the system transmission zeros and the associated input and state directions.
(d) Using your results from (a) and (c), show how transmission zero energy propagates through the system.
(e) Discuss the controllability and observability of the system. ■

Problem 6 (Model Based Compensator Design)

Consider the two input LTI plant P with state space quadruple A_p, $B_p = [\, B_{p_1} \;\; B_{p_2} \,]$, C_p, $D_p = [\, D_{p_1} \;\; D_{p_2} \,]$. Suppose that a $u_{p_1} = \left(\frac{1}{s}\right) u_1$ augmentation is required in the first control channel and a $u_{p_2} = \left(\frac{1}{s^2+100}\right) u_2$ augmentation is required in the second control channel. Sketch a block diagram for the augmented system. Give a state space representation for the augmented system. Discuss how you might complete a model based compensator design based upon the above design plant. ■

Problem 7 (Linearization)

Consider the nonlinear dynamical system $\dot{x} = x^2(1-x) - x^2 u$. (a) Determine the set of all system equilibria. (b) Show that the pair $(x = \frac{1}{4},\, u = \frac{3}{4})$ represents a system equilibrium. (c) Determine an LTI small signal model defined by the state space pair (A, B) that is valid near the equilibrium in (b). (d) Discuss how the linear model can be used to simulate the nonlinear system. ■

Problem 8 (State Space Arithmetic)

Consider a classic negative feedback system with first order SISO LTI plant P, first order SISO LTI controller K, and exogenous signals (r, d_i, d_o, n) where $y = d_o + Pu$, $u_p = d_i + Ke$, $e = r - n - y$. Suppose that the closed loop system has an associated state space representation

$$\begin{bmatrix} \dot{x}_p \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_p \\ x_k \end{bmatrix} + \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} r \\ d_i \\ d_o \\ n \end{bmatrix}. \tag{2}$$

Determine the transfer functions P and K. ■